COMPUTATION AND VISUALIZATION OF REACHABLE DISTRIBUTION NETWORK SUBSTATION VOLTAGE

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ABSTRACT

In this work, we present two visualizations of the controllability of substation voltage in a multiphase distribution network with distributed energy resources (DER). We utilize a linearized unbalanced power flow model to formulate a mathematical program that determines the voltage feasibility and cost (in terms of DER control effort) to move substation voltages. The algorithms are tested via numerical simulations on an IEEE test feeder.

INTRODUCTION

The overall electric grid network is managed by many disparate entities. For example, in California there is both a system operator, mostly concerned about maintaining the delicate balance of load and generation, and utility operators, mostly concerned about the reliability and safety of the lower network levels. Within a utility there are also the distribution operators, and within this mix we consider a diverse set of stakeholders. These stakeholders include the individual customers, and now aggregators of DER to the grid. DER aggregators will in the near future be able to bid their resources into new markets and will be considered an integral part of the generation landscape. This in tandem with the modeling and data integration issues outlined earlier – leads to a conundrum of how to account for these resources and distribution behavior, while working within both human and machine limitations.

Traditionally transmission operators consider the distribution system to be a load, and distribution operations consider transmission an infinite source. With DER integration the former is no longer true, and distribution may become a resource to transmission. Numerous approaches have been proposed to integrate transmission and distribution models into a singular platform. In certain cases, the brute force approach of using high performance computing to simulate larger and larger, and multi-layered, network models is appropriate. However, in the absence of real time large and multi-layer network simulations, transmission operators may require a useful visualization of the performance of a distribution system, and the state to which they can control it at the substation bus (or lower) [1] - [3]. We seek to address this second case providing useful and accurate data to operators on the aggregate performance and controllability of the distribution system, in a simple visualization environment.

In this paper, we present two novel visualizations as metrics of distribution system control performance. The first visualization shows the feasible three phase voltage magnitude at the distribution level substation as a three dimensional shape. The second visualization plots a metric of DER cost to achieve a desired substation voltage profile. To the author's knowledge, no work in this area has been published.

This paper is organized as follows: We first present a linearized unbalanced power flow model. Next, we incorporate the model into a mathematical program, and develop algorithms to visualize substation voltage feasibility and cost. Finally, we discuss results from simulations performed on an IEEE test feeder and provide concluding remarks.

LINEAR THREE PHASE POWER FLOW MODEL

In this section, we outline a linear three-phase power flow model. A derivation of the model can be found in [4], which are a three-phase representation of the *LinDistFlow* equations [5].

Consider a radial distribution feeder $\mathcal{T} = (\mathcal{N}, \mathcal{E})$ where $\mathcal{N}$ is the set of nodes and $\mathcal{E}$ is the set of edges. Edge $(m, n) \in \varepsilon$ connects nodes m and n. A single subscript of m or n denotes the node; a double subscript mn denotes an edge (m, n). A single superscript ϕ denotes a phase from the set $\{a, b, c\}$.

The complex load on phase ϕ at node m is given by:

$$s_m^\phi = s_m^\phi\left(A_{PQ,m}^\phi + A_{Z,m}^\phi \left|V_m^\phi\right|^2\right) + w_m^\phi \quad \textbf{(1)}$$

where $A_{PQ,m}^\phi + A_{Z,m}^\phi = 1$. The complex DER power dispatch is $w_m^\phi = u_m^\phi + jv_m^\phi$.

The conservation of complex power at node m is given by:

$$S_m = s_m + \sum_{n:(m,n)\in\varepsilon} S_n \quad \textbf{(2)}$$

where $S_m = [S_m^a \quad S_m^b \quad S_m^c]^T = P_m + jQ_m \in \mathbb{C}^{3\times 1}$ and

$s_m = [s_m^a \quad s_m^b \quad s_m^c]^T \in \mathbb{C}^{3\times 1}$.

The vector $Y_m \in \mathbb{R}^{3\times 1}$ is comprised of the squared voltage magnitudes of the voltage phasors at node m. The complex scalar V_m^ϕ represents the voltage phasor for phase phi at node m. If phase ϕ does not exist at node m, then $Y_m^\phi = V_m^\phi = 0$.

$$Y_m = \begin{bmatrix} Y_m^a \\ Y_m^b \\ Y_m^c \end{bmatrix} = \begin{bmatrix} |V_m^a|^2 \\ |V_m^b|^2 \\ |V_m^c|^2 \end{bmatrix} \quad \textbf{(3)}$$

The relation between voltage magnitudes between nodes m and n is:

$$Y_m \approx Y_n - M_{mn}P_n - N_{mn}Q_n \quad \textbf{(4)}$$

with M_{mn} and N_{mn} defined as:

$$M_{mn} = \begin{bmatrix} -2r_{mn}^{aa} & r_{mn}^{ab} - \sqrt{3}x_{mn}^{ab} & r_{mn}^{ac} + \sqrt{3}x_{mn}^{ac} \\ r_{mn}^{ba} + \sqrt{3}x_{mn}^{ba} & -2r_{mn}^{bb} & r_{mn}^{bc} - \sqrt{3}x_{mn}^{bc} \\ r_{mn}^{ca} - \sqrt{3}x_{mn}^{ca} & r_{mn}^{cb} + \sqrt{3}x_{mn}^{cb} & -2r_{mn}^{cc} \end{bmatrix} \quad \textbf{(5)}$$

$$N_{mn} = \begin{bmatrix} -2x_{mn}^{aa} & x_{mn}^{ab} + \sqrt{3}r_{mn}^{ab} & x_{mn}^{ac} - \sqrt{3}r_{mn}^{ac} \\ x_{mn}^{ba} - \sqrt{3}r_{mn}^{ba} & -2x_{mn}^{bb} & x_{mn}^{bc} + \sqrt{3}r_{mn}^{bc} \\ x_{mn}^{ca} + \sqrt{3}r_{mn}^{ca} & x_{mn}^{cb} - \sqrt{3}r_{mn}^{cb} & -2x_{mn}^{cc} \end{bmatrix} \quad \textbf{(6)}$$

where $z_{mn}^{\phi\psi} = r_{mn}^{\phi\psi} + jx_{mn}^{\phi\psi}$ is the complex impedance between phases ϕ and ψ on edge *(m,n)*.

VISUALIZATION ALGORITHMS

In this section, we present optimization programs and algorithms designed to visualize the performance and controllability of a distribution network. Subscripts for voltages (e.g. Y_{650}^ϕ) refer to nodes of the test feeder studied in simulations shown in Figure 1.

Algorithm for Finding Distribution Substation Feasible Voltage Magnitude

Our first objective is to determine the set of feasible voltages at the network substation that are reachable given available DER. We formulate a convex optimization program (7) that seeks to minimize the voltage magnitude of phase ϕ at the substation, subject to the linear three-phase power flow model, voltage constraints, and bounds on DER power.

$$\begin{aligned} &\underline{Y_{650}^\phi} = \min_{Y,P,Q,u,v} Y_{650}^\phi \\ &subject\ to: (1) - (\mathbf{6}) \\ &|w_n^\phi| \le \overline{w_n^\phi} \\ &0.95 \le |V_n^\phi| \le 1.05 \end{aligned} \quad \textbf{(7)}$$

$$\begin{aligned} &\overline{Y_{650}^\phi} = \max_{Y,P,Q,u,v} Y_{650}^\phi \\ &subject\ to: (1) - (\mathbf{6}) \\ &|w_n^\phi| \le \overline{w_n^\phi} \\ &0.95 \le |V_n^\phi| \le 1.05 \end{aligned} \quad \textbf{(8)}$$

Similarly, (8) seeks to maximize the same voltage magnitude, subject to the same constraints.

The algorithm to find the feasible substation voltage magnitudes is outlined as follows:

Algorithm 1: Determination of Feasible Voltages

1. Solve 7 and 8 for $\underline{Y_{650}^a}$ and $\overline{Y_{650}^a}$, respectively
2. Grid over **a** phase magnitude from $\sqrt{\underline{Y_{650}^a}}$ to $\sqrt{\overline{Y_{650}^a}}$
3. At each **a** phase grid point A_k, solve 7 and 8 for $\sqrt{\underline{Y_{650}^b}}$ and $\sqrt{\overline{Y_{650}^b}}$, respectively, with the additional constraint that $Y_{650}^a = {A_k}^2$
4. At the current **a** phase grid point A_k, grid over **b** phase from $\sqrt{\underline{Y_{650}^b}}$ to $\sqrt{\overline{Y_{650}^b}}$
5. At each **b** phase grid point B_k, solve 7 and 8 for $\underline{Y_{650}^c}$ and $\overline{Y_{650}^c}$, respectively, with the additional constraints that $Y_{650}^a = {A_k}^2$ and $Y_{650}^b = {B_k}^2$
6. After all grid points are exhausted, find the convex hull of magnitude values.

The convex nature of 7 and 8 ensure that an optimal solution is obtained for every instance. The DER magnitude constraints can also be approximated as set of linear equations, thus making (7) and (8) linear programs.

Algorithm for Determining Substation Voltage Magnitude DER Cost

Our second objective is to determine the approximate cost to achieve a certain voltage magnitude profile at the substation. We formulate a third optimization program in which the cost function is the sum of the magnitudes of all DER output.

$$\begin{aligned} &L(|V_n^a|, |V_n^b|, |V_n^c|) = \min_{Y,P,Q,u,v} \sum_{n\in\mathcal{N},\phi\epsilon\{a,b,c\}} |w_n^\phi|^2 \\ &subject\ to: (1) - (\mathbf{6}) \\ &|w_n^\phi| \le \overline{w_n^\phi} \\ &0.95 \le |V_n^\phi| \le 1.05 \end{aligned} \quad \textbf{(9)}$$

We formulate 9 with a generic objective function of DER power dispatch in lieu of an economic cost. The algorithm to find the optimal cost of achieving a substation voltage magnitude profile is outlined as follows:

Algorithm 2: Optimal Voltage Profile DER Cost

1. Fix **a** phase voltage magnitude to desired value A_D
2. Solve 7 and 8 for $\underline{Y_{650}^b}$ and $\overline{Y_{650}^b}$, respectively, with the additional constraint that $Y_{650}^a = A_D^2$
3. Grid over **b** phase from $\sqrt{\underline{Y_{650}^b}}$ to $\overline{Y_{650}^b}$
4. At each **b** phase grid point B_k, solve 7 and 8 for $\underline{Y_{650}^c}$ and $\overline{Y_{650}^c}$, respectively, with the additional constraints

that $Y_{650}^{a} = A_{D}^{2}$ and $Y_{650}^{b} = {B_k}^2$

5. At the current **b** phase grid point B_k, Grid over **c** phase from $\underline{Y_{650}^{c}}$ to $\overline{Y_{650}^{c}}$
6. At each **c** phase grid point C_k, solve 9 with the additional constraints that $Y_{650}^{a} = A_{D}^{2}$ and $Y_{650}^{b} = {B_k}^2$ and $Y_{650}^{c} = {C_k}^2$

This algorithm is designed to create a cost visualization for a desired **a** phase voltage magnitude, as we consider a color map within a three dimensional shape to be a poor presentation of data. Without loss of generality, this can be done for desired voltage magnitudes on other phases.

Remarks on Algorithms

The algorithms presented grid over the feasible voltage magnitude space. Thus a large number of convex optimization program calls must be completed, which may result in high computation time for larger networks. We are investigating ways to reduce this computation effort.

The feasible substation voltage magnitude can be mathematically defined using polytope projections. The linear power flow model, (1) - **(6)**, can be formulated as the linear matrix equation $Ax \leq b$. The voltage magnitude constrains of $0.95 \leq |V_n^{\phi}| \leq 1.05$ are linear. The DER dispatch magnitude bound can be approximated by a set of linear equations, therefore both sets of constrains from (7) and (8) can be incorporated into $Ax \leq b$.

SIMULATION RESULTS

We perform simulations using a modified version of the IEEE 13 node test feeder. Feeder topology can be seen in Figure 1. The voltage regulator between nodes 650 and 632 and the transformer between nodes 633 and 634 are omitted. The switch between nodes 671 and 692 is assumed closed.

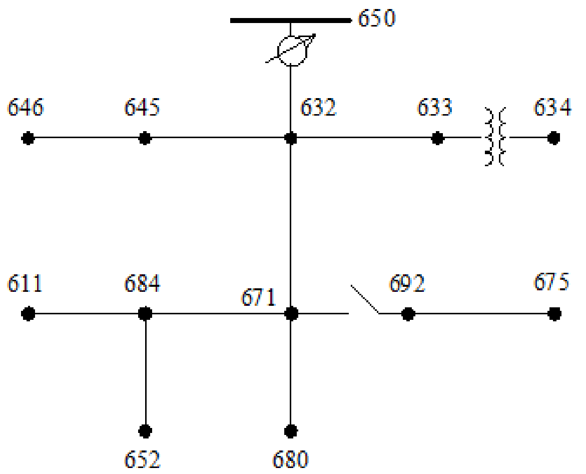

Figure 1: IEEE 13 node test feeder topology

We assume the substation, node 650, is connected to a transmission line which acts as an infinite source. The substation and transmission line are connected by the transformer specified in the test feeder documentation. We assume the cross phase impedances of the transformer are one fifth of the same phase impedance.

Spot loads are multiplied by a factor of 1.125 to produce phase **c** voltage violations in zero DER dispatch. For simplicity all loads, have parameters $A_{PQ,m}^{\phi} = 0.85$ and $A_{Z,m}^{\phi} = 0.15$ for simplicity.

DERs are placed at nodes 632, 671, 692, and 684, on all existing phases at each node. For simplicity, DERs are assumed to be single phase four quadrant operation capable inverters. DER apparent power dispatch is limited to 0.075 p.u. for all inverters.

We assume that the voltages on the transmission line are 1.0 p.u. for phases **a**,**b**, and **c**. Solving power flow, assuming all DER power to be zero, substation voltage magnitudes for phases a,b, and c are 0.9898, 0.9900, and 0.9836, respectively.

Simulations results of the first visualization, showing reachable substation voltage magnitude, are given in Figure 2 - Figure 5. Figure 2 shows the reachable voltage magnitude as a three-dimensional polygon. Figure 3 shows this shape projected onto the **ab** phase plane, Figure 4 shows this shape projected onto the **ac** phase plane, and Figure 5 shows the reachable voltage projected onto the **bc** phase plane.

Figure 2 shows that a voltage magnitude profile of $Y_{650} = [1 \quad 1 \quad 1]^T$ is achievable, showing that with proper DER control, balancing of voltages with limited loss of power is effective with this strategy.

It can clearly be seen in Figure 3, Figure 4, and Figure 5 that the **b** phase maximum voltage magnitude is more constrained than that of **a** and **c**. This may indicate higher loading conditions or less DER availability or penetration.

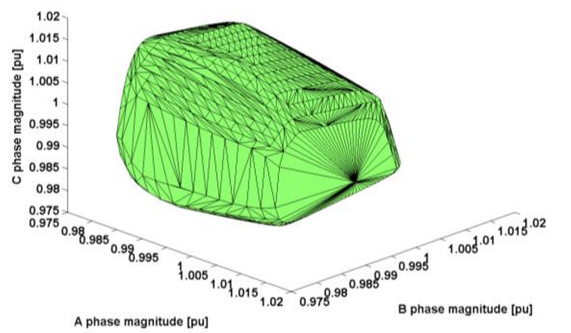

Figure 2: Feasible voltage magnitude at substation

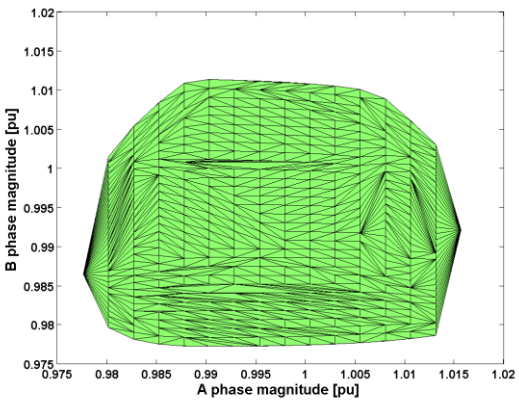

Figure 3: Feasible substation voltage magnitude plotted on AB phase plane

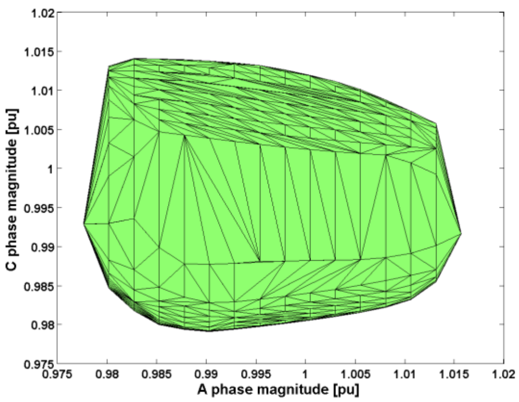

Figure 4: Feasible substation voltage magnitude plotted on AC phase plane

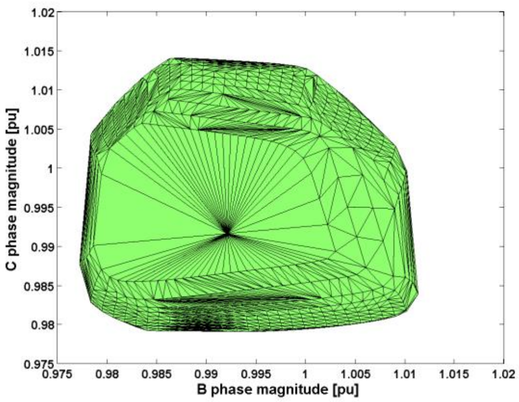

Figure 5: Feasible substation voltage magnitude plotted on BC phase plane

Figure 6 through Figure 9 show the p.u. DER power cost for achieving substation voltage magnitude profiles for four fixed **a** phase magnitudes. The differences in reachable voltage in the **bc** phase plane is clearly visible, as phase **a** magnitude is increased. The DER power dispatch cost is also clearly shown for different desired substation voltages as is the different costs to balance substation voltage at different magnitudes.

Plots such as Figure 2 - Figure 9 may benefit system operators in understanding the state of the network. Additionally, this information may inform operators in their DER control decisions, such how costly it is to balance voltage magnitude at the substation.

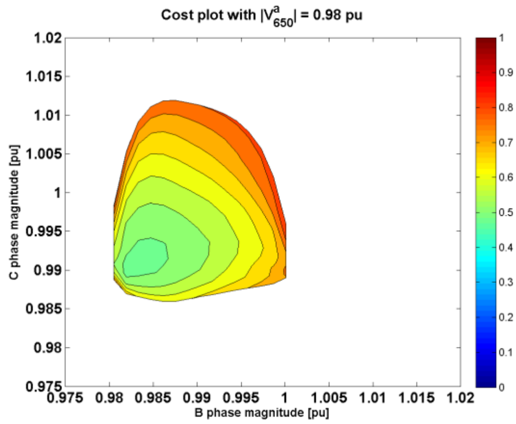

Figure 6: DER p.u. power cost plot for achievable substation voltage magnitude with a phase voltage magnitude of 0.98

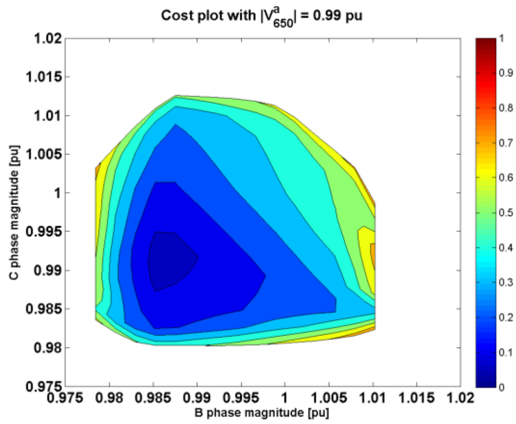

Figure 7: DER p.u. power cost plot for achievable substation voltage magnitude with a phase voltage magnitude of 0.99

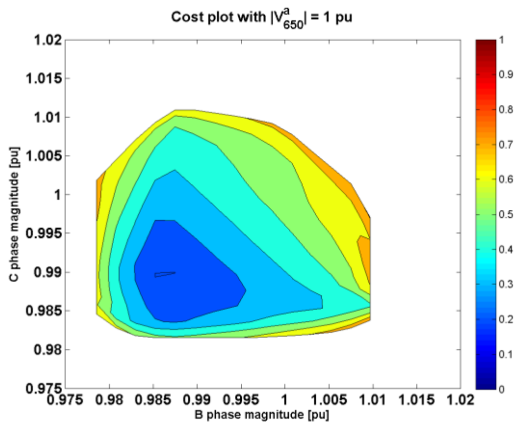

Figure 8: DER p.u. power cost plot for achievable substation voltage magnitude with a phase voltage magnitude of 1.00

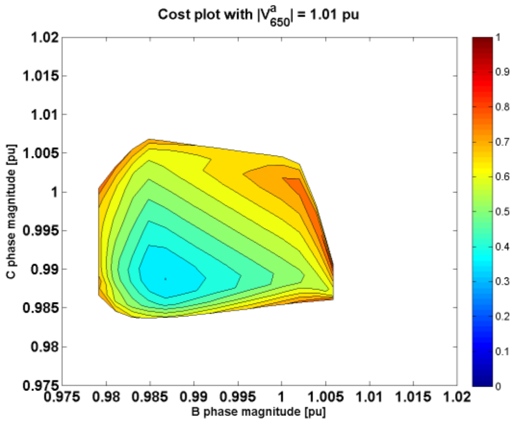

Figure 9: DER p.u. power cost plot for achievable substation voltage magnitude with a phase voltage magnitude of 1.01

CONCLUSIONS

In this work, we present mathematical programs and an algorithm to compute the reachable voltage magnitude of unbalanced radial distribution network substation. We also present an algorithm to calculate the optimal cost of achieving desired substation voltage profile. Simulation results are presented as visual tools for system and utility operators.

We plan to further develop this work in three key areas. The first is to develop more realistic single and three phase inverter models. The second is to improve computation time by exploring polytope representations of network mode, and grid model reduction. The third is to investigate temporal reachability with changing load conditions and energy storage.

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