

## RESOLVING INCONSISTENCIES IN THREE-PHASE CURRENT MEASUREMENTS

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### ABSTRACT

It is widely assumed that any unbalance between phase currents in 4-wire distribution circuits will equal the neutral current; that is that the sum of currents is zero. In practice, however, measurements made at distribution substations often show significant inconsistency. Several possible explanations for this are explored, showing that calculations of the voltage drop and losses are more accurate if harmonics are included. More generally, feeder configurations are described in which the currents may not sum to zero.

### INTRODUCTION

The assumption that the sum of currents equals zero is a key principle used when calculating the impedance of cables and overhead lines. This assumption forms the basis of the reactance calculations used in standard texts by Kersting and Anderson, and is a requirement for the development of Carson's equations [1–3]. However, the validity of this assumption is rarely demonstrated and practical experience shows that measurements in LV networks often appear inconsistent. An example is shown in Fig. 1 for measurements at 1 minute resolution of the phase and neutral currents at an LV substation busbar [4].

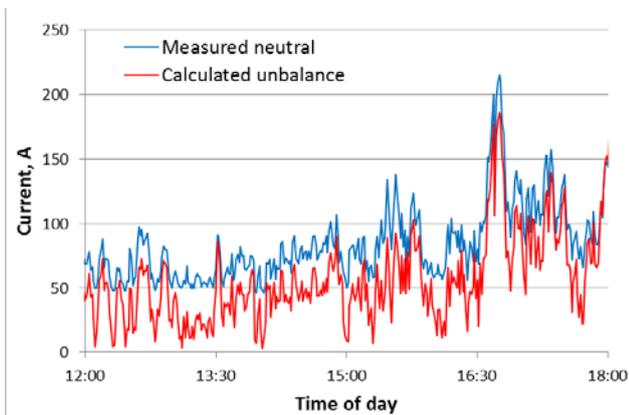


Fig. 1 Measured and calculated neutral current for measured substation data, 1 minute resolution

For a four-wire system, the neutral current would be expected to equal the unbalance current calculated from the phase currents. The unbalance current  $I_U$  is calculated using the RMS phase current amplitudes and corresponding phase angles and is given by:

$$I_U = \left| \sum_{i=1}^3 I_{\text{rms},i} \cdot e^{-j \cdot (2\pi(i-1)/3 + \angle(P_i + jQ_i))} \right| \quad (1)$$

where  $I_{\text{rms},i}$  is the RMS amplitude of phase  $i$ . The real and reactive power  $P_i$ , and  $Q_i$  are used to determine the phase angle of the current relative to the voltage. Ideal 120° voltage phase angles are assumed.

At some times of the day Fig. 1 shows relatively close agreement between the predicted unbalance and the measured neutral current but there are also periods in which the two values are significantly different.

This paper aims to identify the causes of this inconsistency and then to propose means by which the effects on calculations using measurement data can be minimised.

### POTENTIAL EXPLANATIONS

The apparent differences could potentially relate to the measurement methodology, possibly due to:

- **Time resolution.** Measurement data typically provides the RMS amplitude averaged over a measurement interval (1 minute in Fig. 1). Although the phase currents may appear balanced when averaged over this period, the mean neutral current may be non-zero if short-term variations of the phase currents within the averaging period are unbalanced.
- **Harmonics.** The calculation in (1) assumes that the measured RMS currents represent the amplitudes of sinusoids at the fundamental frequency but the measured RMS includes any harmonics present.
- **Measurement errors.** Noise or systematic errors.

It is also possible that the measured conductors do not contain all of the current. This could be due to:

- **Neutral conductor loops.** Although LV networks generally have a radial topology the neutrals typically remain inter-connected at link boxes.
- **Ground currents.** The analysis conventionally used to allow for the ground path in the cable impedance assumes that the current follows the cable route. In practice, the ground is a continuous conductor and the current may follow more direct paths.

### Time resolution

Fig. 2 examines the current data for the above example in further detail. This shows the maximum and minimum values that occurred within the 1 minute averaging periods for the phase L2 and neutral currents. It is clear that there is significant variation that the 1 minute resolution does not capture.

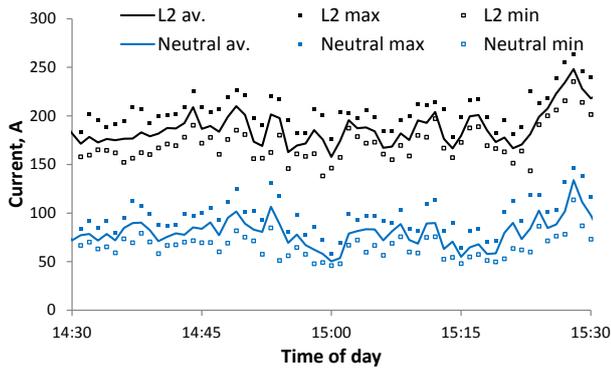


Fig. 2 Average, maximum and minimum currents for measured substation, 1 minute resolution

## ANALYSIS OF TEST MEASUREMENTS

The measurement effects have been investigated using data recorded for an LV feeder cable at Loughborough University. This forms a network spur with no link boxes at the feeder end. The feeder has TN-S grounding such that the neutral is separated from the cable screen and from the earth bonding. In this case, all of the unbalance current should therefore return via the neutral (except in fault conditions). The feeder was lightly loaded, serving office circuits and also a three-phase variable speed pump. The combined load is unbalanced and highly distorted with up to 40% current THD on the most heavily loaded phase conductor.

Monitoring data was recorded with a Fluke 435-II power quality analyser, capturing the RMS current and voltage at 250 ms resolution. The logs also included amplitudes and phases of all harmonics up to the 21<sup>st</sup>, recorded at the same resolution.

The RMS phase and neutral currents are shown in Fig. 3 for a selected period, together with the unbalance current calculated using (1). This shows a similar (although less severe) disparity to that in Fig. 1.

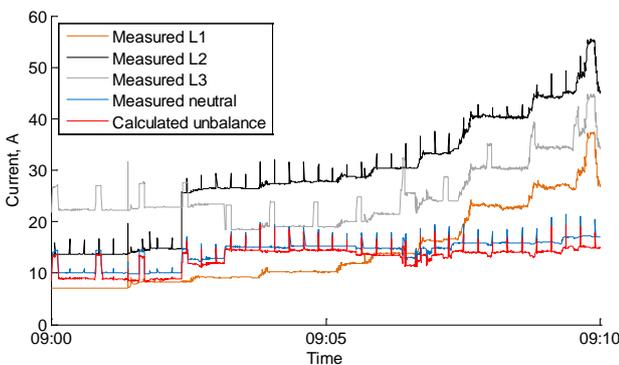


Fig. 3 RMS current amplitude and calculated unbalance

## Time resolution

In order to investigate the effect of time resolution, the logging data has been post-processed to synthesize the

current data that would have been obtained if a longer averaging period had been used. Fig. 4 shows the difference between the neutral and unbalance currents at resolutions of 1 minute and 250 ms. The plot also shows the maximum and minimum values that would be recorded with 1 minute averaging. At 250 ms resolution, the maximum and minimum values have negligible deviation from the average. For this example, an increase in resolution to 250 ms is sufficient to give a good representation of the time variation of the phase and neutral currents. However, the difference between the neutral and unbalance currents remains.

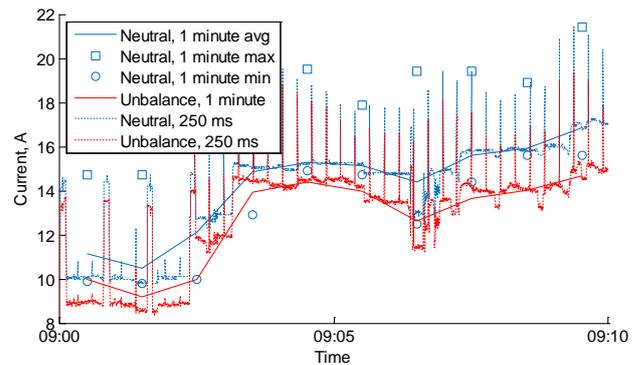


Fig. 4 Effect of time resolution

## Harmonics

In (1) the unbalance current is calculated on the basis that the RMS current in each phase represents the amplitude of a sinusoid at 50 Hz. In practice it represents the sum of contributions at the fundamental frequency and from all of the harmonics. If the harmonic data is available, an improved calculation of the unbalance current can be made by considering each frequency separately. A phasor value is formed for each phase conductor and the unbalance current is then calculated for each harmonic frequency. The measured RMS neutral current is then expected to equal the square root of the sum of squares of the unbalance contributions from each frequency.

This comparison is shown in Fig. 5 and demonstrates a much better agreement between the measured neutral and the calculated unbalance using the harmonic data.

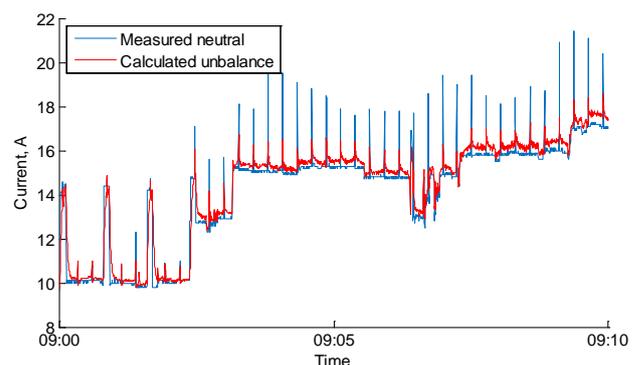


Fig. 5 Neutral current and unbalance using harmonics

Some slight differences remain and it is assumed that these are due to amplitude and phase tolerances in the Rogowski coil current sensors, accurate to  $\pm 1\%$  in amplitude (with  $\pm 2\%$  for positioning) and  $\pm 1^\circ$  phase [5].

## MITIGATION OF MEASUREMENT EFFECTS

The test data has been used to explore the impacts of not accounting for harmonics. Calculations of the voltage drop and losses with the full harmonic data included have then been compared with simpler estimates that consider a single frequency model at 50 Hz. These alternatives use define the amplitude of the 50 Hz sinusoid according to either the measured RMS amplitude or an estimate of the amplitude of the fundamental component.

The feeder uses a 120 m length of 4-core 95 mm<sup>2</sup> cable to BS 5467 design with no intermediate junctions. The 3×3 phase impedance matrix with terms  $\hat{Z}_{h,ij}$  has been calculated for each frequency, allowing for AC resistance effects and the frequency-dependent self- and mutual reactances [6].

The measurement data has been used to calculate the voltage drop and losses along the test cable length. With harmonics included, the voltage difference  $\Delta V_{h,i}$  with current phasor  $I_{h,i}$  in phase  $i$  and harmonic  $h$  is:

$$\Delta V_{h,i} = \sum_{k=1}^3 \hat{Z}_{h,ik} I_{h,k} \quad (2)$$

This voltage difference is subtracted from the measured voltage phasors  $V_{h,i}$  to calculate the voltage phasors at the downstream end of the cable. At each node, the RMS voltage of the time domain waveform is equal to the square root of the sum over all frequencies of squares of the phasor amplitudes. The voltage drop  $V_{d,i}$  is then given by the difference between the RMS voltages for each end of the cable, given by:

$$V_{d,i} = \sqrt{\sum_{h=1}^{N_h} |V_{h,i}|^2} - \sqrt{\sum_{h=1}^{N_h} |V_{h,i} - \Delta V_{h,i}|^2} \quad (3)$$

where  $N_h = 21$  is the number of frequencies.

In the absence of harmonic data, the calculation is based on a sinusoid at the fundamental frequency. One option for defining the current is to use the measured RMS amplitude. The phase angles are derived from the voltage and power factor as in (1). This voltage difference is:

$$\Delta V_{1,i} = \sum_{k=1}^3 \hat{Z}_{1,ik} \cdot I_{\text{rms},k} \cdot e^{j(\angle V_{1,k} - \angle (P_k + jQ_k))} \quad (4)$$

The voltage drop is then calculated using (3) but with  $\Delta V_{h,i} = 0$  for  $h > 1$ .

An alternative approach is to use the amplitude of the fundamental phasor component  $|I_{1,i}|$  alone. This can be estimated from THD data which is often available from

logging instruments even if the full harmonic data is not recorded. The amplitude of the fundamental component is then given by:

$$|I_{1,i}| = I_{\text{rms},i} / \sqrt{1 + (\text{THD}_i/100)^2} \quad (5)$$

The voltage drop is then calculated as above using (4) and (3) but with  $|I_{1,i}|$  in place of  $I_{\text{rms},i}$ .

Fig. 6 compares the voltage drop for these three cases. The calculation in which the current is taken as the amplitude of a sinusoid at 50 Hz (denoted ‘RMS’) over-estimates the voltage drop by up to approximately 7% compared to the more accurate model with the harmonics included. The voltage drop calculation with the fundamental alone is a slight over-estimate but is much closer to the result with harmonics included. In the absence of the full harmonic data, this third option provides a more accurate voltage drop calculation than using the measured RMS to define the current amplitude.

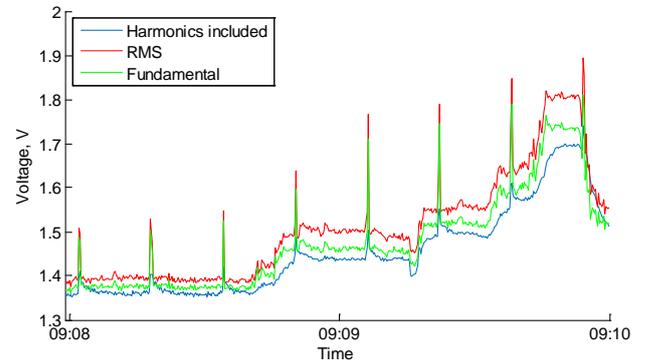


Fig. 6 Voltage drop for phase L2

A similar comparison can be made for the losses. With harmonics included, the loss power is given by:

$$P_{\text{loss}} = \text{re} \left( \sum_{i=1}^3 \sum_{h=1}^{N_h} (\Delta V_{h,i} \cdot I_{h,i}^*) \right) \quad (6)$$

The results for the three cases described above are shown in Fig. 6. The losses are under-estimated with both single frequency methods with an error of up to 28% if the current is defined using the amplitude of the fundamental. However, the error is reduced to below 9% by using the measured RMS to define the current amplitude.

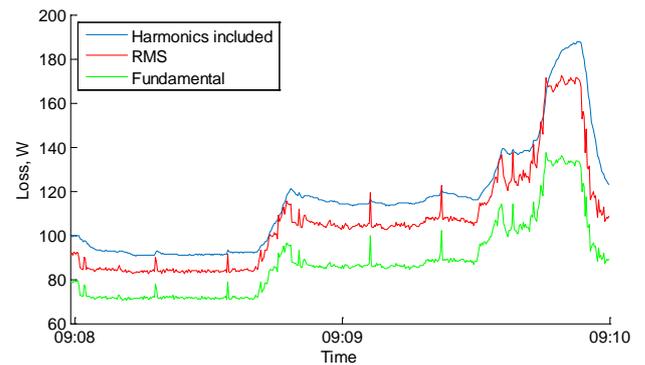


Fig. 7 Loss power

## ADDITIONAL CONDUCTIVE PATHS

The cable for the tests described was carefully selected as an example of a feeder with no further connections to the neutral, and so that the measured currents were expected to sum to zero. More generally, there are additional conductive paths that may need to be considered.

### Neutral conductor loops

The neutral conductors in LV networks often have permanent connections at link boxes. Loop paths can exist between feeders connected to the same busbar or between feeders from different substations to allow for redundancy [7, 8].

In Fig. 7, link A allows for circulating currents in the loop created in the LV feeders from substation 1. This does not affect measurements at the substation busbar but currents in individual feeders may not sum to zero.

Link boxes B and C connect feeders from substations 1 and 2. If only one of these link boxes exists then circulating currents are blocked by the DY configuration of the transformers which do not pass zero sequence currents via the three-wire HV network. However, if both B and C exist, then circulating currents may occur in the loop between both substations.

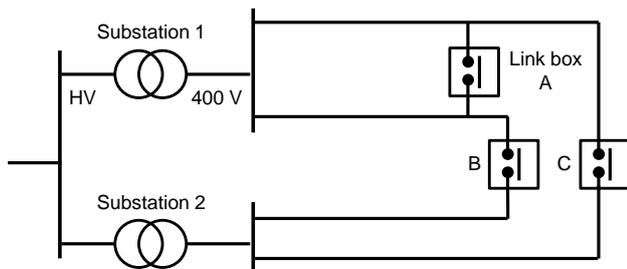


Fig. 8 LV feeder interconnections

### Ground currents

The neutral conductors in networks with protective multiple earthing (PME) are connected to ground electrodes at feeder ends and via earth bonding at customer connections. There is therefore a conductive path through the ground to the electrode at the substation. This is typically included into the impedance calculations using Carson's equations [1, 3] which model the ground as if it were an additional conductor running alongside the cable or overhead line. The ground resistivity is of the order of  $10^9$  times greater than that of the cable and so the ground conductor implied by Carson's equations has a low current density that is spread over a distance of around 1 km [6]. This presents a problem if Carson's equations are applied to LV networks where the space between feeders is much less than this distance.

Fig. 8 shows some additional conductive paths that may exist between ground electrodes. One possibility is that

unbalance current entering the ground at the end of a long feeder may take a 'short cut' to the substation rather than follow the route of the cable. A second possibility is for currents to circulate in a loop between the ground electrodes of nearby substations. The currents in the substation busbars may then not sum to zero.

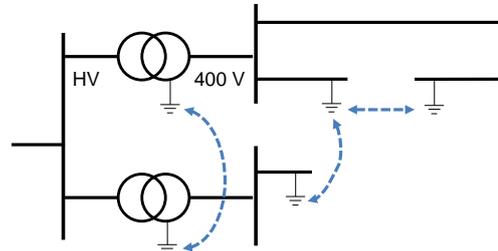


Fig. 9 Ground conductor possible paths

The impact of these possible ground current paths and of the neutral loops described above needs further investigation through simulation and measurement.

## CONCLUSIONS

Current measurements on LV networks can appear inconsistent such that the sum of currents in all of the conductors is non-zero. The causes and effects of these inconsistencies have been investigated using measurements taken from a real LV feeder on the Loughborough University campus.

Harmonics were found to be the main cause of the inconsistency in the measured data. Summing the currents in the usual way (considering a single frequency system at 50 Hz) led to significant inconsistency, whereas summing the harmonic frequencies individually was much better.

Analysis with a single frequency model over-estimated the voltage drop by 7% if the measured RMS was used to represent the amplitude of a sinusoid at 50 Hz. A better estimate was obtained by using THD information to find the amplitude of the fundamental current component. Conversely, losses were better approximated for a 50 Hz model using the measured RMS amplitudes.

Temporal averaging (the use of 1-minute data which smooths over the rapid variations) was shown not to be a major cause of inconsistency in calculating the unbalance current, although the higher resolution reveals more detail of the demand variation.

More generally, a number of additional current paths exist through neutral conductors connected at link boxes and between ground electrodes in PME networks. These effects are not typically taken into account in network models and require further investigation.

## ACKNOWLEDGEMENTS

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