EVALUATION OF EXTENDED KALMAN FILTER AND PARTICLE FILTER APPROACHES FOR QUASI-DYNAMIC DISTRIBUTION SYSTEM STATE ESTIMATION

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ABSTRACT
Monitoring of distribution systems plays an important role in the enhancement of today’s distribution systems for future requirements. Tasks of distribution system operation are becoming more active and the demand for network observability increases. In this paper two innovative methods for quasi-dynamic distribution system state estimation, the Extended Kalman Filter and the Particle Filter approach, are evaluated regarding their usability for distribution system operation. Both have been applied to the medium voltage CIGRE benchmark network and are investigated for different situations of measurement configurations and pseudo-measurement modelling.

INTRODUCTION
Since the amount of distributed generation (DG) is steadily growing, monitoring methods become increasingly important for distribution system operators (DSO). Power system state estimation (SE) has been implemented and proved for high voltage (HV) transmission system operation in the past decades [1]. SE is used to monitor the current system state based on redundant measurements and topology information supplied by SCADA systems. Recent research discusses the application of distribution system state estimation (DSSE) to enhance observability on lower voltage levels caused by the demand for active management of distribution systems. The authors in [2] provide a survey on the current state of the art regarding DSSE. The main reason traditional SE cannot be transferred to distribution system operation easily is the lack of redundant measurement data. Several papers discuss the generation of pseudo-measurements (PM) to complete input data for DSSE, proposing methods like Artificial Neural Networks [3] or Gaussian Mixture Model for load estimation [4]. Another frequently discussed aspect is the optimal placement of measurement devices to reduce the absolute number of required measurements with minimal loss of information [5] [6].

The estimation method considered in most applications and investigations on DSSE are performed with the Weighted Least Squares (WLS) approach which is known from traditional SE [1]. In [7] different estimators, for example Weighted Least Absolute Value and Schweppe Huber generalised M, are discussed and evaluated for DSSE. However, alternative estimation methods for DSSE in general have not been discussed extensively in research. In [8] and [9] the Kalman Filter method and the enhancement to Extended Kalman Filter (EKF) are proposed to incorporate time-history data in estimation problem. In [9] it is shown that the EKF methods leads to higher accuracy in estimation results compared to WLS and also improves convergence behaviour. Since this approach seems to be promising, further investigations are made in this paper to show the benefits of the EKF on the one hand, and to outline the limits of this approach on the other hand. The structure of the EKF approach indicates that it is not robust in presence of lower accuracy in input data, especially in non-linear systems [8]. Another limitation is the precondition of Gaussian-distributed errors in the EKF approach, which may not be valid for PM in DSSE [9].

To overcome the problems of robustness and non-Gaussian-distributed errors a Particle Filter (PF) approach is explored and compared to the EKF approach in this paper. The PF has been introduced in several fields of research [10], but not in context of DSSE by now. One application in the context of power systems can be found in the dynamic state estimation [11]. Dynamic state estimation refers to the estimation of variables in transient time range, for example frequency or generator speed. In contrast static or quasi-dynamic state estimation aims to identify the operating point of the network in steady state. The term “quasi-dynamic” also indicates that not only a snapshot but a time series is considered.

This paper evaluates both approaches for quasi-dynamic DSSE. At first the basic principles of state estimation methods are introduced, followed by a presentation of the implemented algorithms. The EKF and PF approaches are tested in different case studies with varying measurement configurations and PM modelling. The advantages and constraints of each approach are emphasized and guidelines for the application in DSSE are deduced.

STATE ESTIMATION APPROACHES
The objective of state estimation is to determine the complex voltages at all buses, since with those a system is fully specified and the current operating state is identified. Therefore, a given set of measurements and topological network information are being processed with a state estimation approach, most commonly a WLS approach. The measurement model relates the measurements z to the system state x by the nonlinear function h:

\[ z = h(x) + e. \]

(1)
The variable $e$ contains the difference between the measurement $z$ and the true value which is unknown. The difference $e$ is often referred to as error or residual. Depending on the accuracy of different measurements, each value $z_i$ is related to a standard deviation $\sigma_i$. The measurement error covariance matrix $R$ is defined as a diagonal matrix with $\sigma_i^2$ on the main diagonal and all elements off the main diagonal equal to zero. The objective function of the WLS approach is to minimize the weighted sum of squared residuals $f(x)$:

$$f(x) = \sum_{i=1}^{m} \frac{(z_i - h(x))^2}{R_{ii}} = \text{min.}$$ \hspace{1cm} (2)

The solution for $x$ can be obtained with an iterative scheme, for example Gauss-Newton method, including the calculation of $H$, being the Jacobian matrix of $h$.

**Extended Kalman Filter**

The EKF is an extension of the linear Kalman Filter to nonlinear estimation problems. The algorithm is implemented as in [9] according to the following iterative procedure with iteration index $k$:

1. Prediction of the state vector $x$ and the calculation of the state covariance matrix $S$:

$$x_{k+1} = \hat{x}_k + J_{PF}^k [u_{k+1} - u_k] + w_k.$$ \hspace{1cm} (3)

$$\hat{S}_{k+1} = \hat{S}_k + Q.$$ \hspace{1cm} (4)

2. Calculation of the Kalman Gain $K$:

$$K_{k+1} = H\hat{S}_{k+1} [H\hat{S}_{k+1} H^T + R_{k+1}]^{-1}.$$ \hspace{1cm} (5)

3. Obtain the measurements $z$ and update the state estimate $\hat{x}$:

$$\hat{x}_{k+1} = x_{k+1} + K_{k+1} [z_{k+1} - Hx_{k+1}].$$ \hspace{1cm} (6)

4. Update the covariance matrix $S$:

$$S_{k+1} = [I - KH]\hat{S}_{k+1}.$$ \hspace{1cm} (7)

For the first time step in 1. the prediction of the system state $x_k$ is equivalent to initializing the state, for example with a flat start, while the covariance matrix $S_k$ is set to $Q$. The prediction step includes calculating the power flow Jacobian $J_{PF}$ to extrapolate the system state for the next time step based on the difference of power balances $u_k = [P_1 \ldots P_n Q_1 \ldots Q_n]^T$ with the prediction $u_{k+1}$. The input covariance is represented in $w_k$.

**Particle Filter**

The PF is a recursive Bayesian approach to the state estimation problem. The implementation for the following investigations is also found under the name bootstrap filter. It is one of the most basic realisations of PF and applicable to a large class of models [10]. The algorithm is implemented according to the following procedure as in [10]:

1. Initialisation by generation of random samples $x_k(i) = 1, \ldots, N_{\text{samp}}$ from the PDF $p(x_k|D)$ with $D$ as a presumed distribution of $x_k$.

2. Evaluation of the normalised importance weights based on received measurement $z_k$ and the likelihood of each sample with

$$q_i = \frac{p(z_k|x_k^*(i))}{\sum_{j=1}^{N_{\text{samp}}} p(z_k|x_k^*(j))}.$$ \hspace{1cm} (8)

3. Resampling from the discrete distribution $\{x_k(i); i = 1, \ldots, N_{\text{samp}}\}$ $N_{\text{samp}}$ times to generate samples $\{x_k(i); i = 1, \ldots, N_{\text{samp}}\}$ with

$$P_r = \{x_k(i) = x_k^*(i)\} = q_i$$ as the posterior distribution.

To obtain the PF state estimate $x$ the mean of posterior distribution is calculated. Other characteristics, not only a single point estimate, of the posterior distribution could be extracted if more detailed investigations are necessary.

**CASE STUDY**

In order to evaluate both state estimation methods several variations for measurement configurations and PM modelling are investigated. The following sections describe the network and the variations for the case study regarding the measurement configuration and the assumptions about PM.

**Network model and measurement configurations**

The network topology is based on the European 20kV medium voltage (MV) CIGRE benchmark network [12] with a high penetration of DG as illustrated in Figure 1.

![Figure 1: Topology of the European MV CIGRE benchmark network [12] with additional data sources](image-url)
For each bus in Figure 1 an underlying low voltage (LV) network is modelled with predominantly PV feed-in. In this paper different configurations for the measurement infrastructure are investigated to determine the influence on DSSE performance. The different configurations comprise variations across the accuracy and the local distribution of measurement devices. The accuracy is considered as a measurement error which is added to the reference power flow result. The measurement error is modelled according to a normal distribution. Under the condition that 99.99% of the measurement errors lie within the range of the assumed maximum error, the standard deviation corresponds to one quarter about the mean value.

The following assumptions about maximum errors of measurement devices are made according to [13]:

- HV/MV substation and MV measurements: voltage magnitude and current measurements have the lowest error of 1%. Since power is calculated as the product of voltage and current, power measurements have an error of 2%.
- LV measurements: if LV measurements are active, the bus voltage is measured on LV level and the value has to be calculated for the MV level. Therefore, the LV voltage measurements are assumed with an error of 4%. Furthermore, the power measurements, predominantly smart meter, have an error of 10%.

In order to evaluate the influence of measurement accuracy on DSSE results, three different configurations are tested. Table 1 gives an overview of the configurations.

### Table 1: Measurement configurations

<table>
<thead>
<tr>
<th>Measurement configuration</th>
<th>Quantity of MV measurements</th>
<th>Quantity of LV measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>low</td>
<td>high</td>
</tr>
<tr>
<td>M2</td>
<td>medium</td>
<td>medium</td>
</tr>
<tr>
<td>M3</td>
<td>high</td>
<td>low</td>
</tr>
</tbody>
</table>

The measurement devices at the HV/MV substation and the bus at MV side of the transformers (hence bus 1, 2, and 13) are considered as available for all three configurations. For all other buses the number of measurement devices is varied as in Table 1. A low quantity corresponds approximately to 20%, medium to 50% and high to 80% in relation to the total number of buses.

### Modelling of pseudo-measurements

Since the measurement infrastructure on MV and LV level is often not sufficient for the common network calculation methods, it is possible to compensate missing input data with PM as power measurements. In [6] the dimension of measurement error is investigated for PM. It can rise up to 100% in presence of high PV penetration. Therefore, in this paper the pseudo-measurement error is varied between 10%, 50%, and 100%. Furthermore, an example for a non-Gaussian distribution of PM error is evaluated. A generalised extreme value (GEV) distribution according to [14] has been applied with the shape parameter $\xi$ set to 0.09. No voltage measurements are active despite the substation measurements, since voltage measurements would improve the estimation results significantly even though high uncertainty due to PM is assumed. To outline the effect on the state estimates for a worst case measurement configuration, the minimum number of measurements is active.

### Simulation scenario and results

The test scenario consists of a quasi-dynamic simulation for 24 hours with a 15 minute step size. The power flow results for each time step are collected in terms of complex voltages for all buses. The resulting system states in form of voltage magnitudes are exemplarily illustrated for bus 7 due to the fact that the maximum deviations occur at this point in the network. Due to the assumed volatile behaviour of PV feed-in high gradients are generated and they undergo a change of sign several times. Firstly the influence of measurement configuration is evaluated and secondly the effect of PM errors.

Figure 2 demonstrates the voltage magnitude at bus 7 for measurement configurations M1, M2, and M3 for both approaches. The black dotted line represents the reference value from quasi-dynamic power flow calculations.

![Figure 2: Voltage magnitude at bus 7 for M1, M2, and M3 with (a) EKF and (b) PF approach](image-url)
As could be expected, the estimation results for both approaches have a higher accuracy if more MV than LV measurements are available. However, the maximum deviation occur in M1 with only 0.5% for EKF and 0.65% for PF compared to the load flow reference, which may be an acceptable accuracy for DSOs. Thus the enhancement of MV distribution systems might be possible with measurement devices on LV side and, therefore, more cost efficient. The total estimation errors are illustrated in Figure 3.

The results from Figure 4 demonstrate that the EKF approach is much more sensitive to PM errors than the PF approach. For a PM error of 100% the maximum deviation of 4% occurs at hour 13:45 (see also Figure 4.a) simultaneously with the highest gradient. Considering common DSO voltage limits of ±10%, the accuracy of the EKF estimate can be evaluated as insufficient. This effect is caused by the basic premise of Gaussian-distributed measurement and noise in the EKF algorithm. The total estimation error is demonstrated in Figure 5.

Figure 3: Estimation error at bus 7 with different measurement configurations for EKF and PF

In Figure 4 the voltage magnitude at bus 7 for different assumptions about PM errors is shown.

Figure 4: Voltage magnitude at bus 7 for varying PM error with (a) EKF and (b) PF approach

Figure 5: Estimation error at bus 7 subject to PM error for EKF and PF

The development of the total estimation error, in terms of sum of squared residuals, suggests that the error for the EKF approach increases disproportionate compared to the error of the PF approach. Consequently, the PF approach should be preferred if high uncertainty is expected in PM for quasi-dynamic DSSE and no measurement devices are available. The results for the PF approach in Figure 4.b show a systematical deviation upwards compared to the load flow reference, which is caused by the initial distribution for particle generation. Therefore, it can be concluded that a dynamic adjustment for particle generation should be considered for future implementations of PF approaches to improve the estimation quality. Another parameter for adjustment is the sample size $N_{\text{samp}}$ which is set to 1000 in this paper. A higher sample size will lead to better estimation results, but at the cost of computational efficiency. As stated before the analysed case study in Figure 4 and Figure 5 does not contain any voltage measurements close to the evaluated bus. The results underline the importance of measurement data with high accuracy for the EKF approach, especially in situation with high gradients caused by fluctuation through DG.

Figure 6 illustrates the results for EKF and PF approaches in presence of non-Gaussian distributed pseudo-measurement errors.
The restriction of the EKF approach primarily being suitable for Gaussian-distributed measurement errors becomes clear in Figure 6. For GEV-distributed PM errors the accuracy of the EKF approach is inadequately while the PF approach provides better results.

**CONCLUSION AND OUTLOOK**

The increasing demand for active distribution systems places high requirements on estimation methods for DSSE. Two innovative estimation approaches have been proposed and evaluated. For the EKF approach it has been proven that it delivers estimation results with high quality as long as the input data is sufficient. Particular caution is necessary in presence of strong fluctuation, for example caused by DG, in combination with highly uncertain input data, especially if non-Gaussian errors may occur. A basic variant of the PF approach has been applied and also provides acceptable results but with lower accuracy than the EKF approach. Instead the PF approach is more robust for uncertain input data. Efforts should be made in further development of PF approaches with respect to the specific challenges of DSSE. This includes the investigation of statistical properties of PM and the adjustment of tuneable parameters of the PF algorithm.

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**REFERENCES**


