LINEAR THREE-PHASE STATE ESTIMATION FOR LV GRIDS USING PSEUDO MEASUREMENTS BASED ON APPROXIMATE POWER DISTRIBUTIONS

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ABSTRACT

The large increase of distributed generation (DG) of photovoltaic (PV) systems in low voltage (LV) grids results in increasing voltage magnitudes and line loadings. Due to a lack of network observability in present LV grids, distribution system operators (DSOs) cannot detect and respond to any limit violations. Even with data collected by the expected rollout of smart meters, full network observability will not be achieved. Thus, the network state will not be completely known. LV-State Estimation (SE) in combination with pseudo-value generation can provide a way to determine the required network states with respect to voltage magnitudes and line loadings. This paper presents a linear three-phase LV-SE approach computed in phase sequences. The performance of the presented SE approach was investigated on a realistic grid model for different cases with and without pseudo-values. The performance is promising and the approach can be used to provide DSOs with input data for control actions to avoid limit violations and thus will contribute to the further integration of DG.

INTRODUCTION

The distributed generation (DG) of photovoltaic (PV) systems in low voltage (LV) grids is growing enormously. In Germany, nearly 98 % of all installed PV systems are connected to LV grids [1]. This corresponds to 60 % of the total installed power of around 40 GW [2]. Similar values are present for the whole of Europe [3]. As a result of the nowadays high DG in LV grids, the voltage magnitudes and line loadings are increasing. Limit violations already occur and will occur even more frequently with the further increase of DG. Due to a lack of network observability in present LV grids, distribution system operators (DSOs) cannot detect and respond to any limit violations. With the expected rollout of smart meters, some data from LV grids will be collected, but not enough to achieve network observability. Thus, the network state is not fully known. LV-State Estimation (SE) in combination with active and reactive power pseudo-values provides a way to determine the required network states with respect to limit violations. The paper presents a new LV-SE approach and determines the performance of the approach for different cases with and without pseudo-values on a realistic LV grid model. For the performance evaluation, the maximum errors for voltage magnitudes and line loadings have been determined, as they need to be considered for avoiding limit violations, thus calculating on the safe site. First a brief review of existing LV-SE approaches is given. After that, the mathematical background of the new LV-SE is shown. Then, the testbed used to determine the performance of the LV-SE for different cases with and without pseudo-values is presented. Finally, the performance is determined and a conclusion is given. It has been confirmed that a LV-SE based only on voltage measurements and pseudo-values shows good performance to detect voltage violations of upper limits.

STATE OF THE ART

Low voltage-State Estimation

LV-SE should meet the following conditions: (i) Be applicable for three-phase calculation. LV grids are in general unbalanced. Symmetric LV-SE is not able to detect single-phase limit violations. Thus, symmetric LV-SE does not calculate on the safe side. (ii) Provide results always. For the case of limit violations, the results of LV-SE is the input data for triggering control actions as generation or demand side management. Taking the high number of LV grids into consideration, control actions cannot be done by operators, they have to be done closed loop. Thus, LV-SE always needs to provide results. Standard SE algorithms use iterative methods, as the SE problem is by definition nonlinear. These algorithms can lead to convergence problems, thus not providing results. Hence, it is better to use a less accurate method (such as a linearized method), that always provide results. (iii) Be highly computationally efficient. It would be too expensive to install high-performance computers in each low voltage grid. The LV-SE algorithm has to be as efficient as possible.

In the field of SE for distribution systems, a lot of research was done in recent years. A good review of different SE methods for distribution systems is given in [4]. However, most of the methods are not focusing on LV grids. In this paper, only the methods suggested for LV grids will be discussed. Most of the suggested methods are based on the weighted least square (WLS) approach, as in [5], [6] and [7]. In [4] a three-phase SE method using hybrid particle swarm optimization is presented. The method meets condition (i), but fails condition (ii) and (iii), as for the case study 1000 iterations were necessary. The suggested method in [5] is focusing on meter placement, but fails to meet any of the above-mentioned conditions. A linear three-phase LV-SE approach computed in symmetrical components has been presented in [6] and [7]. It meets all conditions, but condition (iii) only partly as for each time step all measurements first need to be transformed into symmetrical components and after the SE calculation transformed back to phase sequences. To overcome these drawback, a linear three-phase SE approach computed in phase sequences has been developed. The new SE approach is also based on the WLS method, but applies a linear Taylor approximation of the measurement functions [8].

Pseudo-values for low voltage grids

In [9] it was shown that accurate pseudo-values for PV system can be obtained if one PV system in the area of the LV grid is measured. Hence, pseudo-values for PV systems will not be investigated in this paper. Methods for the generation of pseudo-value for loads can be categorized in (a) Methods based on load profiles, (b) Probabilistic methods and (c) Advanced mathematical methods.

Methods based on load profiles have the drawback that values for all loads at the same time step usually are similar, what in reality is not the case. Such a method is presented in [10]. Probabilistic methods assume that load values follow a certain probability distribution. The pseudo-values are obtained from these distributions. Such a method is presented in [11]. Advanced mathematical methods are all methods that use extensive calculations. They are usually not efficient enough for grid operation. Such a method is presented in [12]. The method uses artificial neural networks for the pseudo-value generation. A separation of loads into different types (households, industry, electric vehicles etc.) is not discussed, as each method can be applied for all types of loads. The focus of this paper is on the pseudo-values for household loads, as they are by far the most common in LV grids.

NEW LINEAR THREE-PHASE STATE ESTIMATION APPROACH

Mathematical background

The idea behind SE is to calculate the statistically most probable values for a set of state variables, based on a set of measurements with whom they are related. The SE problem is defined by [13] as solving the equation for the measurement model (1) where **z** is the measurement vector of *m* independent measurements, **x** the state vector of *n* variables, **h** the measurement function vector relating **x** to **z** and **e** the error vector of measurements.

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e} \tag{1}$$

In power systems, the state vector is defined as the voltage magnitudes and angles of all nodes. The measurement vector and the measurement function vector depend on the existing measurements. The measurements can be divided into measurements of voltage and current magnitudes and active, reactive and apparent power. Voltage angles can also be measured with phasor measurement unit (PMUs), but this is too expensive for low voltage grids and will not be further considered. The measurement model is nonlinear as all of the measurement types, except the voltage magnitudes, are nonlinear functions of x. The measurement model can be linearized if Taylor series of the measurement functions are applied. Unfortunately, a straightforward linearization of measurement functions for apparent power and current magnitudes is not possible. An approach for their linearization is given in [14]. In this paper, they are not considered further. Assuming the symmetrical component parameters of a three-phase line connecting node *i* with node *j* is known, the admittance matrix of the line in symmetrical components $\underline{\mathbf{Y}}_{012}^{ij}$ can be built. This matrix can be transformed into the admittance matrix in phase sequences $\underline{\mathbf{Y}}_{123}^{ij}$. Now the relation between complex current flows and node voltages can be expressed with (2).

$$\begin{bmatrix} \underline{I}_1^{ij} \\ \underline{I}_2^{ij} \\ \underline{I}_3^{ij} \end{bmatrix} = \underline{\mathbf{Y}}_{123}^{ij} \cdot \left(\begin{bmatrix} \underline{U}_1^{i} \\ \underline{U}_2^{i} \\ \underline{U}_3^{i} \end{bmatrix} - \begin{bmatrix} \underline{U}_1^{j} \\ \underline{U}_2^{j} \\ \underline{U}_3^{j} \end{bmatrix} \right) \tag{2}$$

The complex power flow for phase $v \in [1,2,3]$ is given with (3), where $\underline{Y}_{v,w}^{ij}$ is the element of $\underline{\mathbf{Y}}_{123}^{ij}$ at row *v* and column *w*.

$$\underline{S}_v^{ij} = \underline{U}_v^i \cdot \left(\underline{I}_v^{ij}\right)^* = \underline{U}_v^i \cdot \sum_{w=1}^{3} \left[\underline{Y}_{v,w}^{ij} \cdot \left(\underline{U}_w^i - \underline{U}_w^j \right) \right]^* \tag{3}$$

With the introduction of (4), where φ_v^i is the voltage angle for node *i* and phase *v*, the active and reactive power flow for phase *v* are given with (5) and (6), respectively. $G_{v,w}^{ij}$ and $B_{v,w}^{ij}$ are the conductance and the susceptance of the admittance matrix $\underline{\mathbf{Y}}_{123}^{ij}$ at row *v* and column *w*, respectively. As it can be seen, the equations are nonlinear.

$$\begin{gathered} \Delta\varphi_{v,w}^{ij} = \varphi_v^i - \varphi_w^j, \quad v, w \in [1,2,3] \\ c_{v,w}^{ij} = \cos\left(\Delta\varphi_{v,w}^{ij}\right), \quad s_{v,w}^{ij} = \sin\left(\Delta\varphi_{v,w}^{ij}\right) \end{gathered} \tag{4}$$

$$\begin{aligned} P_v^{ij} = \; & U_v^i \cdot \sum_{w=1}^{3} \Big\{ U_w^i \cdot \left[G_{v,w}^{ij} \cdot c_{v,w}^{ii} + B_{v,w}^{ij} \cdot s_{v,w}^{ii} \right] - \ldots \\ & U_w^j \cdot \left[G_{v,w}^{ij} \cdot c_{v,w}^{ij} + B_{v,w}^{ij} \cdot s_{v,w}^{ij} \right] \Big\} \end{aligned} \tag{5}$$

$$\begin{aligned} Q_v^{ij} = \; & U_v^i \cdot \sum_{w=1}^{3} \Big\{ U_w^i \cdot \left[G_{v,w}^{ij} \cdot s_{v,w}^{ii} - B_{v,w}^{ij} \cdot c_{v,w}^{ii} \right] - \ldots \\ & U_w^j \cdot \left[G_{v,w}^{ij} \cdot s_{v,w}^{ij} - B_{v,w}^{ij} \cdot c_{v,w}^{ij} \right] \Big\} \end{aligned} \tag{6}$$

Assuming the evaluation point (7) and applying Taylor approximation on (5) and (6), the linearized approximation for the active and reactive power flow for phase *v* can be expressed after some mathematical derivations as (8) and (9), respectively.

$$\begin{gathered} \forall U \approx U_{\mathrm{ref}}, \\ \Delta\varphi_{v,w}^{ii} \approx \Delta\varphi_{v,w}^{ij} \approx \Delta\varphi_{v,w,\mathrm{ref}}^{ij} = \begin{cases} 0 & \text{if } (v-w)=0 \\ -2\pi/3 & \text{if } (v-w)=-2 \vee 1 \\ 2\pi/3 & \text{if } (v-w)=-1 \vee 2 \end{cases} \end{gathered} \tag{7}$$

$$P_v^{ij} \approx \sum_{w=1}^{3} U_{\text{ref}} \cdot \left[\left(G_{v,w}^{ij} \cdot c_{v,w,\text{ref}}^{ij} + B_{v,w}^{ij} \cdot s_{v,w,\text{ref}}^{ij} \right) \cdot \left(U_w^i - U_w^j \right) \right] - \dots$$
$$\sum_{w=1}^{3} U_{\text{ref}}^2 \cdot \left[\left(-G_{v,w}^{ij} \cdot s_{v,w,\text{ref}}^{ij} + B_{v,w}^{ij} \cdot c_{v,w,\text{ref}}^{ij} \right) \cdot \left(\varphi_w^i - \varphi_w^j \right) \right] \quad (8)$$

$$Q_v^{ij} \approx \sum_{w=1}^{3} U_{\text{ref}} \cdot \left[\left(G_{v,w}^{ij} \cdot s_{v,w,\text{ref}}^{ij} - B_{v,w}^{ij} \cdot c_{v,w,\text{ref}}^{ij} \right) \cdot \left(U_w^i - U_w^j \right) \right] - \dots$$
$$\sum_{w=1}^{3} U_{\text{ref}}^2 \cdot \left[\left(G_{v,w}^{ij} \cdot c_{v,w,\text{ref}}^{ij} + B_{v,w}^{ij} \cdot s_{v,w,\text{ref}}^{ij} \right) \cdot \left(\varphi_w^i - \varphi_w^j \right) \right] \quad (9)$$

For U_{ref}, e.g. the nominal voltage of the grid or the voltage at the transformer can be chosen. The expressions can be extended for power injections, as shown with (10). Where N is the number of grid nodes.

$$P_v^i = -\sum_j^N P_v^{ij} \quad Q_v^i = -\sum_j^N Q_v^{ij} \quad (10)$$

With the linearization, all measurements are linear functions of the state vector, thus the measurement model (1) can be expressed with (11), where **H** is the measurement function matrix, formed based on (8), (9) and (10).

$$\mathbf{z} = \mathbf{H} \cdot \mathbf{x} + \mathbf{e} \quad (11)$$

With the application of WLS, the estimated state $\hat{\mathbf{x}}$ is given with (12), where **R** is a diagonal matrix of the corresponding measurement variances [13].

$$\hat{\mathbf{x}} = \left(\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} \right)^{-1} \mathbf{H}^T \mathbf{R}^{-1} \mathbf{z}, \quad \mathbf{R} = \text{diag}\left(\sigma_1^2, \dots, \sigma_m^2 \right) \quad (12)$$

Application of pseudo-values

A necessary condition for the application of SE is a positive redundancy η. This means, the number of measurements must be higher than the number of state variables, as seen from the redundancy definition (13).

$$\eta = \frac{m}{n} - 1, \quad n = 3 \cdot (2 \cdot N - 1) \quad (13)$$

As mentioned before, the number of measurements in LV grids are too low and positive redundancy cannot be achieved with measurements only. Hence, pseudo-values for loads are necessary. In this paper, the application of pseudo-value generation based on approximate power distributions for loads was tested. The generation process is presented in [11]. The results of [11] show, that for each time step the distribution of power values for all loads in a LV grid can be approximated with sufficient accuracy and good properties for use with the LV-SE.

TESTBED

Model of the LV grid

The LV-SE approach was tested on the actual models of the LV grids from the projects SmartSCADA and CheapFlex. Information about the projects can be found in [7] and [11]. In this paper the results for the CheapFlex grid are presented. Fig. 1 shows the CheapFlex grid. The grid has 258 nodes, 260 lines, 128 loads and 18 PV systems. It is supplied with a 400 kVA-transformer with eight feeders.

Figure 1. Aerial view of the LV grid from the CheapFlex project, the transformer position is indicated.

Time series for loads and photovoltaic systems

Three-phase time series for loads and PV systems have been gathered in a 10-minute measurement interval during a two-year measurement campaign on around 100 households within the SmartSCADA project. The time series for active and reactive power for a 1-year period from 1st January 2015 till 1st January 2016 were used.

Benchmark

The time series were randomly assigned to the loads and PV systems of the LV grid. With the assigned values, power-flow calculations were performed. The results of the power-flow calculations are used as a benchmark for the estimated results.

Test cases

Four different cases were defined to determine the performance of the LV-SE approach. Tab. 1 gives an overview of the investigated cases. For each case, the values or the structure of the measurement vector **z** are different. In other words, the assumed measurement scope is different. A separation between the transformer node (TR), household nodes (HH) and zero injection nodes (ZI) was made. Zero injection nodes are grid connection nodes without power injection or consumption. In LV grids, they are mostly underground nodes without the possibility to be measured. The subscript B+σ means that input measurements are chosen as equal to the corresponding benchmark values superimposed with $N(0,\sigma^2)$-noise. The values for σ were chosen based on the accuracy of smart meters. These values are the representation of real measurements. The subscript AD means that input

Table 1. Overview of investigated test cases

	Case 1		Case 2		Case 3		Case 4	
Input for **z**	Input	$\{\sigma\}$	Input	$\{\sigma\}$	Input	$\{\sigma\}$	Input	$\{\sigma\}$
	$P_{\text{B+}\sigma}^{\text{TR}}$	1	$P_{\text{B+}\sigma}^{\text{TR}}$	1	$P_{\text{B+}\sigma}^{\text{TR}}$	1	$P_{\text{B+}\sigma}^{\text{TR}}$	1
	$P_{\text{B+}\sigma}^{\text{HH}}$	1	$P_{\text{B+}\sigma}^{\text{HH}}$	1	$P_{\text{B+}\sigma}^{\text{HH}}$	1	$P_{\text{AD}}^{\text{HH}}$	60
	$P_{\text{B+}\sigma}^{\text{ZI}}$	1	$P_{\text{B+}\sigma}^{\text{ZI}}$	1	$P_{\text{B+}\sigma}^{\text{ZI}}$	1	$P_{\text{B+}\sigma}^{\text{ZI}}$	0.5
	$Q_{\text{B+}\sigma}^{\text{TR}}$	1	$Q_{\text{B+}\sigma}^{\text{TR}}$	1	$Q_{\text{B+}\sigma}^{\text{TR}}$	1	$Q_{\text{B+}\sigma}^{\text{TR}}$	1
	$Q_{\text{B+}\sigma}^{\text{HH}}$	1	$Q_{\text{B+}\sigma}^{\text{HH}}$	1	$Q_{\text{B+}\sigma}^{\text{HH}}$	1	$Q_{\text{AD}}^{\text{HH}}$	60
	$Q_{\text{B+}\sigma}^{\text{ZI}}$	1	$Q_{\text{B+}\sigma}^{\text{ZI}}$	1	$Q_{\text{B+}\sigma}^{\text{ZI}}$	1	$Q_{\text{B+}\sigma}^{\text{ZI}}$	0.5
	$U_{\text{B+}\sigma}^{\text{TR}}$	0.1	$U_{\text{B+}\sigma}^{\text{TR}}$	0.1	$U_{\text{B+}\sigma}^{\text{TR}}$	0.1	$U_{\text{B+}\sigma}^{\text{TR}}$	0.1
	$U_{\text{B+}\sigma}^{\text{HH}}$	0.1	$U_{\text{B+}\sigma}^{\text{HH}}$	0.1	$U_{\text{B+}\sigma}^{\text{HH}}$	0.1	$U_{\text{B+}\sigma}^{\text{HH}}$	0.1
	$U_{\text{B+}\sigma}^{\text{ZI}}$	0.1	$U_{\text{B+}\sigma}^{\text{ZI}}$	0.1	$U_{\text{B+}\sigma}^{\text{ZI}}$	-	$U_{\text{B+}\sigma}^{\text{ZI}}$	-
	$\forall\varphi_U$	-	$\forall\varphi_U$	0.01	$\forall\varphi_U$	0.01	$\forall\varphi_U$	0.01
$\eta \approx$	1.5		2		1.75		1.75	

Units of measurements: $[P]$ = 'W', $[Q]$ = 'var', $[U]$ = 'V', $[\varphi]$ = 'rad'.
Superscript meaning: TR – Transformer node, HH – Household node, ZI – Zero injection node.
Subscript meaning: B+σ – noisy benchmark values, AD – values from approximate distributions.

measurements are pseudo-values chosen randomly from the approximate active power distribution with a power factor of 0.9 from [11]. Their values for σ are equal to the assumed pseudo-value accuracy. If there is no value for σ in a row, the corresponding measurements do not occur in **z**. The symbol $\forall\varphi_U$ means that all voltage angles were applied to **z**. If this is the case, the voltage angles of each node are assumed equal to the voltage angle of the transformer (slack) node. Case 1 represents the situation where the voltage magnitude, active and reactive power values of all nodes are measured. The purpose of this case is to investigate the sole accuracy of the new LV-SE approach. Case 2 represents an extended case 1. The only difference to case 1 is that all voltage angles occur in **z**, with the assumption that all voltage angles are the same as the slack voltage angle. This leads to more redundancy, but also to some error. The purpose of this case is to investigate this error. Case 3 represents a case that really can be achieved with measurement equipment. The voltage of the zero injection nodes can in most case not be measured, as they are usually underground. About half of the nodes in a LV grid are zero injection nodes, so the redundancy is reduced by 0.25 (one-quarter of the number of state variables). The purpose of this case is to investigate what would be the accuracy of the approach if smart meters would measure voltage magnitudes, active and reactive powers of all households. Case 4 is a modified case 3. In this case, the active and reactive power values for households are replaced with pseudo-values. The reason for this case is, that in Germany it is only allowed to measure voltage magnitudes for households [15].

RESULTS

As mention before, the maximum errors for voltage magnitudes and line loadings are of interest. They are calculated by (14). Where the subscript "Case" indicates the case values and "Benchmark" the benchmark values.

$$U_{\text{error}} = U_{\text{Case}} - U_{\text{Benchmark}} \;,\; I_{\text{error}} = I_{\text{Case}} - I_{\text{Benchmark}} \qquad (14)$$

Tab. 2 shows the error range for the different cases over all time steps. A separation between the maximum positive and the maximum negative values was made to find out if the LV-SE approach has a certain tendency. Another reason for this separation is that today DSOs are interested in the detection of higher voltage magnitudes or line loadings. For this detection, positive errors are not in the scope of interest as they lead to results on the safe side. In the future, when automatic control actions will be performed in LV grids, also the lower voltage magnitudes and line loading can be in the scope of interest. For this reason, the whole error range was investigated. It can be seen that cases 1, 2 and 3 have very similar and small errors. This leads to the conclusion that the LV-SE algorithm without pseudo-values is accurate. For the case with pseudo-values (case 4), an acceptable value of 1.3 V for the maximum negative voltage magnitude error and an acceptable value of 27.25 A for the maximum positive line loading error was calculated. The maximum positive voltage magnitude error and the maximum negative line loading error are too high and not acceptable. The reason for this is that the used pseudo-value generation has a limit for active power pseudo-values of 2 kW. In reality, higher values may occur, for example for households with heat pumps. Thus the pseudo-values are too small and lead to noticeable smaller calculated voltage drops and line loadings. The same can be noticed with Fig. 2, where the range of voltage magnitude and line loading errors for different daytimes for case 3 and 4 are shown. The highest errors occur for time steps with high loads. The pseudo-value generation has to be improved to generate active power values higher than 2 kW. Another option is to apply bad data detection and in this way find loads with too small values. First tests with bad data detection showed promising results.

CONCLUSION

The paper presents a novel LV-SE approach computed in phase sequences. The performance of the approach is excellent if all loads of a LV grid are measured. As this is

Table 2. Error range for the different cases

Case:	1	2	3	4
$\max(U_{\text{error}})$	0.917 V	0.925 V	0.969 V	5.724 V
$\lvert\min(U_{\text{error}})\rvert$	0.386 V	0.383 V	0.400 V	1.301 V
$\max(I_{\text{error}})$	3.724 A	3.741 A	3.801 A	27.250 A
$\lvert\min(I_{\text{error}})\rvert$	3.362 A	3.296 A	3.398 A	43.254 A

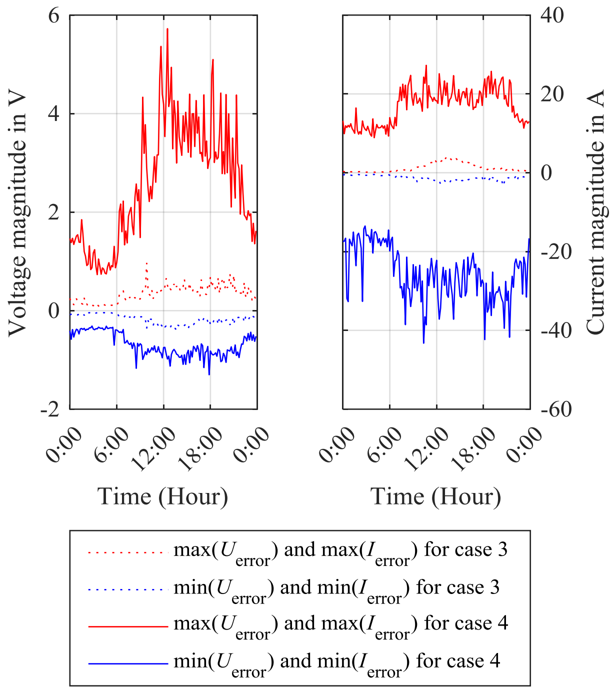

Figure 2. Range of voltage magnitude and line loading errors for different daytimes for case 3 and 4.

never the case, the approach was also tested with the use of pseudo-values. The pseudo-value generation based on approximate power distributions for loads was chosen. The results with pseudo-values show that the pseudo-value generation needs to be improved for higher loads or a bad data detection needs to be applied to get applicable results. Nevertheless, as the maximum negative voltage error is an acceptable 1.3 V, the approach can at least be used to detect voltage violations of upper limits caused by DG.

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REFERENCES

[1] Fraunhofer Institute for Solar Energy Systems ISE, "Aktuelle Fakten zur Photovoltaik in Deutschland," Report, Freiburg, October 2016.

[2] Deutsche Gesellschaft für Sonnenenergie e.V., "Die Karte der Erneuerbaren Energien," Accessed (December 2016) from: http://www.energymap.info.

[3] M. Vandenbergh et al., "Prioritization of technical solutions available for the integration of PV into the distribution grid," Report, June 2013.

[4] S. Nanchian, A. Majumdar and B. Pal, "Three-phase state estimation using hybrid particle swarm optimization," *2016 IEEE Power and Energy Society General Meeting (PESGM)*, Boston, MA, 2016.

[5] A. Abdel-Majeed, S. Tenbohlen, D. Schöllhorn and M. Braun, "Meter placement for low voltage system state estimation with distributed generation," *22nd International Conference and Exhibition on Electricity Distribution (CIRED 2013)*, Stockholm, 2013.

[6] D. Waeresch, R. Brandalik, W. H. Wellssow, J. Jordan, R. Bischler and N. Schneider, "State estimation in low voltage grids based on smart meter data and photovoltaic-feed-in-forecast," *23rd International Conference on Electricity Distribution (CIRED 2015)*, Lyon, 2015.

[7] D. Waeresch, R. Brandalik, W. H. Wellssow, J. Jordan, R. Bischler and N. Schneider, "Linear state estimation in low voltage grids based on smart meter data," *2015 IEEE Eindhoven PowerTech*, Eindhoven, 2015.

[8] A. Cate, "Simultaneous One-Step Approximations of Real and Reactive Power Flow," *Journal of Power and Energy Engineering 2*, 2014, pp. 28-33.

[9] M. Cramer, P. Goergens, F. Potratz, A. Schnettler and S. Willing, "Impact of three-phase pseudo-measurement generation from smart meter data on distribution grid state estimation," *23rd International Conference on Electricity Distribution (CIRED 2015)*, Lyon, 2015.

[10] M. Hasheminamin, V. G. Agelidis and A. Heidari, "Impact study of high PV penetration in low and medium-voltage networks when considering residential and industrial load profile," *2013 International Conference on Renewable Energy Research and Applications (ICRERA)*, Madrid, 2013, pp. 347-352.

[11] R. Brandalik, D. Waeresch and W. H. Wellssow, "Approximate active power distributions for standard household loads," *2016 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, Beijing, China, 2016.

[12] A. Abdel-Majeed, C. Kattmann, S. Tenbohlen and R. Saur, "Usage of Artificial Neural Networks for pseudo measurement modeling in low voltage distribution systems," *2014 IEEE PES General Meeting | Conference & Exposition*, National Harbor, MD, 2014.

[13] A. Abur, *Power System State Estimation*, CRC Press, Boca Raton, USA, 2004.

[14] D. Waeresch, R. Brandalik, W. H. Wellssow, J. Jordan, R. Bischler and N. Schneider, "Bad data processing for low voltage state estimation systems based on smart meter data," *CIRED 2016 Workshop: Electrical networks for society and people*, Helsinki, 2016.

[15] German Federal Ministry of Economic Affairs and Energy, "Gesetz zur Digitalisierung der Energiewende (English: The Act on the Digitisation of the Energy Transition)", Berlin, August 2016.