

## NONLINEAR OPTIMAL CONTROL OF THE RESPONSES OF RESIDENTIAL SMALL HOUSES WITH NEW ENERGY RESOURCES

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### ABSTRACT

*Dynamic demand response is increasingly needed and dynamic retail tariffs are available in Finland. Heat pumps and solar panels increasingly affect electricity consumption of houses. Thermal and electricity storages offer potential to reduce electricity costs by providing flexibility to the power system. This paper analyses the potential of nonlinear optimisation in dynamic price control of such resources in the houses.*

### INTRODUCTION

The subject addressed is nonlinear dynamic optimal price control of residential houses that have heat pumps, heat storing floor heating, battery energy storage (BES) and solar power generation. The control signal may be the sum of the day-ahead price and the distribution tariff, or it may be a price determined by the electricity retailer or aggregator in a way agreed with the consumer to take into account other markets, balancing, and distribution network issues. The responses to the price signal must be automated, because only timely, reliable and predictable responses are useful for the electricity markets, emission mitigation and power distribution.

The suitability and benefit potential of nonlinear optimal control for the houses is studied. An own MATLAB implementation of the generalised reduced gradient optimisation and the principle of Pontryagin is used for this. The nonlinear optimal control enables appropriate formulation of the optimisation criterion. It can also take into account nonlinear BES characteristics and the partial dimensioning of the heat pump and its nonlinear coefficient of performance. The nonlinearities of the BES introduce to the optimisation problem severe challenges for the applied method such as multiple optima and severe discontinuities of the gradient, when switching between charging and discharging. There are earlier reports on the application of nonlinear optimal control to active houses, such as [1],[2],[3], [4] and [5].

Two test houses had been modelled using measurements. A nonlinear heat pump model was added to these models of the thermal dynamics of the houses. A ground source heat pump was installed in one of the houses and measurements from it were used to tune and verify the model parameters. Full 4 week periods in four different seasons are studied. In the method the weather forecast and weather measurements are used to forecast the thermal load and solar power generation. The price signal was the sum of static time-of-use distribution tariff and the dynamic area day-ahead spot market price. Feed-in to

the network was allowed with the spot market price and taking into account the small maximum allowed distribution tariff for feed in to the network. A penalty term was applied to mitigate rapid load variations typically occurring around the change of the hour in the optimal solutions. With the method such penalty terms and hard constraints or additional price terms can easily be used for taking into account possible network constraints etc.

The benefit potential from optimal control for heat pump houses with solar power and electricity storage and how it depends on dimensioning is studied by simulations. In addition also the pros and cons of nonlinear optimal control as compared to linear optimal control and some heuristics and including network based constraints in the optimisation are considered.

### METHODS

#### Dynamic thermal balance models

Models for the dynamic heat balances of the buildings were developed based on preliminary information on the buildings and measurements made during 2004-2015. MATLAB System Identification Toolbox was used. The models are linear except for constraints and the heat pump coefficient of performance. Also ventilation rate affects nonlinearly in the model, but now it was kept constant.

Ground source heat pump with water circulation and a photovoltaic panel were added to the model used in [1]. One of the two houses was already included in that study. The state variables are the following lumped temperatures:

- temperature of the indoor air
- temperature of internal walls
- temperature of the outside walls
- temperature of the heat storing floors
- temperature of the heat storing fireplace
- temperature of the sauna
- temperature of the hot water storage
- temperature of the circulating heating water.

The main uncontrollable input variables are outdoor air temperature, solar radiation and occupancy. Occupancy affected via the usage of appliances and hot domestic water. The controllable inputs were electrical powers to 1) the heat pump, 2) direct electrical heating via the heating element of the heat pump system and 3) the storing floor heating. The heat pump system takes only the sum of 1) and 2) as input and internally always prioritises the using of the heat pump. The losses of the circulation pump are modelled as a minimum limit for the direct heating power. Domestic hot water is heated by the

heat pump system.

### Battery energy storage models

Battery energy storages (BES) comprising an inverter and a battery was added in the house energy balance model described above. Two types of batteries were modelled based on the data given by the manufacturers: 1) a typical lead-acid battery [6] and 2) a modern Li-Titanate (LTO) battery [7]. The impact of the BES purchase costs (200€/kWh for the lead-acid, 1100€/kWh for the LTO) via the accelerated battery aging by usage was modelled only separately and not included in the optimisation models. Such costs of aging by usage were so high for the lead-acid battery are nearly as high as the benefit from the battery. For the LTO battery modelled the aging by this usage was insignificantly small (between 0.005-0.025€/kWh with assumed high cycle lifetime). Only the model of the LTO battery and its results are described here.

A polynomial was fit to the battery energy storage efficiency data, because the optimisation method assumes continuous smooth functionals, see Fig. 1.

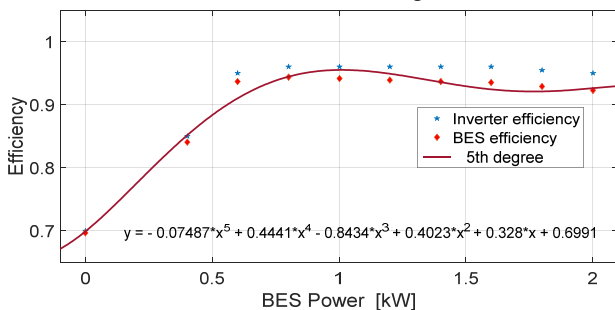


Fig. 1. The model of BES efficiency.

In addition the model includes the LTO battery's quadratic dependence of the losses to the electrical power.

The battery charging and discharging were modelled separately and also the state of charge was split to two tightly connected state variables. This was necessary in order to make the model better conditioned for the optimisation. The optimisation method as such is not able to handle well the severe discontinuities in the gradient that locally suggest that changing the direction of the current would bring negative losses.

### Forecasting the solar power production

Finnish Meteorological Institute (FMI) open weather forecast data [8] are utilized in the photovoltaic production forecasting algorithm. Implementation is presented in [9]. Sun position angles are modelled according to [10] and used to calculate the solar radiation on a tilted surface. It comprises direct, diffuse and reflected radiation. The radiation on tilted panel surface is calculated with the HDKR model (the Hay, Davies, Klucher, Reindl model). More information is in [3].

### Nonlinear constrained optimisation of dynamic control

Determining the best response of the house to price variations is an optimisation task, where the objective is to minimise power purchase costs while maintaining comfortable indoor conditions. A nonlinear constrained optimization method was previously developed for the purpose and implemented in MATLAB. The method is based on the generalized reduced gradient method with the gradient calculated from the adjoint state using the principle of Pontryagin. The approach is explained in detail in [11].

Time step  $dt = 10$  minutes was used in the optimisations. In the simulations four optimisation period of four weeks were used. With a laptop PC each 16 week optimisation case including the four compared methods took less than 1 hour. For on-line spot price control optimisation a period that covers two days is usually sufficient and the additional benefit from longer periods is rather small.

With minor simplifications the optimisation problem formulation is

$$\begin{aligned}
 x(t+dt) &= f_i(x(t), u(t), w(t)), \quad x(t_0) = x_0, \\
 y(t) &= Dx(t), \\
 u_{\min}(t) &\leq u(t) \leq u_{\max}(t), \\
 p_{\text{house}}(t) &= \text{sum}(u(t)) + Ew(t) + p_{\text{module}}(t), \\
 J &= \sum (f_2(p_{\text{house}}(t)) + (x(t) - x_{\text{des}}(t))^T Q (x(t) - x_{\text{des}}(t))). \quad (6)
 \end{aligned}$$

Where  $x$  is for each time point  $t \in [t_0, t_{\max}]$  the state vector comprising temperatures and battery state of charge,  $u$  the controlled heating powers and BES charging and discharging powers,  $u_{\min}$  and  $u_{\max}$  the control constraints,  $w$  the non-controllable inputs,  $f_i$  the thermal balance function,  $y$  the measurement vector,  $D$  the measurement matrix,  $Ew$  the non-controlled power consumption,  $p_{\text{module}}$  the solar power production,  $p_{\text{house}}$  the power of the house,  $Q$  the matrix weighting state deviation from the desired state  $x_{\text{des}}$ ,  $f_2$  the electricity cost, and  $J$  the optimisation criterion summed over the time period of optimisation.

When using the method occasional failures to converge were observed. This was completely solved by generating five different initial guesses with other methods and taking the best solution reached. The methods for producing the initial guesses included no-price control base case, two differently tuned heuristic price control approaches, and two Time of Use methods.

The main advantage of the nonlinear optimisation method is that it allows formulating nonlinear criteria. With linear optimisation such modelling that uses the storage both before and after the price peak is difficult and sensitive to model changes. With heating and cooling loads a quadratic criterion describes the problem better than a linear one and gives solutions with more benefit. Also

nonlinearities due to heat pump coefficient of performance and dimensioning are easy to model. As such the method is not yet able to handle clumped start-up and shutdown cost. Integrating to a mixed integer approach and detailed modelling of start-ups and shut-downs are obvious potential alternative solutions.

### Adjusting the forecast based solution by feedback from the current state

The optimisation was based on early morning forecasts. Thus the forecasting errors affect the solution and the indoor temperatures may fluctuate outside the acceptable region. Thus simple feedback from the state of the building was applied to adjust the indoor temperatures towards those that the forecast based solution would have given, if the actual would have been according to the forecasts. In addition feedforward was included to add self-consumption to heating in case of actual solar power production exceeded the forecast one. These adjustments are still under development. Improving the forecast accuracy or the feedback structure and tuning will move the results towards the optimisation with perfect forecast case that is also calculated as an estimate of maximum potential benefit.

### Distribution grid constraints in the optimisation

Time variable distribution grid constraints are included in the optimisation model in three ways. The maximum power of the house is both limited and associated with a quadratic cost. Also the electricity price can be changed.

## SIMULATION TEST CASES

### Electricity prices

The variable electricity costs for the consumers were as in Helsinki in 2016 with Nord pool spot market day-ahead prices. Time of use distribution tariff was applied and all the taxes included, see Fig. 2. The distribution tariff for feed-in was assumed to be the maximum allowed. The electricity retailer was assumed to buy back the feed-in with the spot price. Only the retailer margins were ignored.

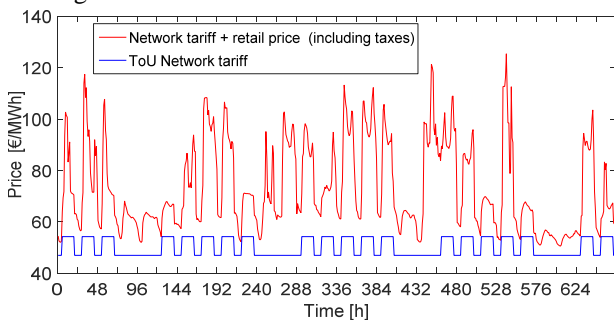


Fig. 2. Electricity price to the customer in April 2016

### Solar power generation and forecast

The forecast and measured solar power productions are shown in Fig. 3.

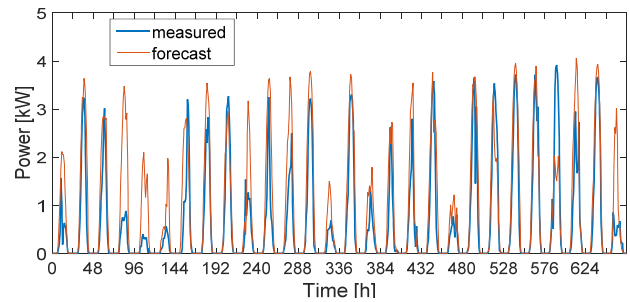


Fig. 3. Forecast and measured solar power production for a PV system with nominal power 5 kW in April 2016.

### Outdoor temperature and its forecast

The forecast and measured ambient temperatures were in April 2016 as in Fig. 4. They are open data by FMI.

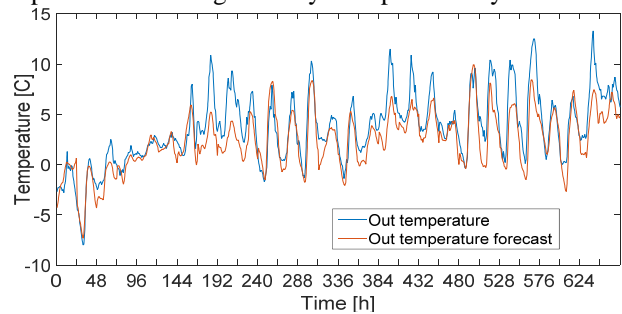


Fig. 4. Forecast and measured outdoor temperature.

### Simulated houses

Two typical detached houses in Helsinki were measured, modelled and simulated. The internal volume of one house is 500 m<sup>3</sup> and the other is 640 m<sup>3</sup>. The heat demands of the houses are very similar. The bigger house is older and it now has a ground source heat pump. In the simulations both houses use the same model of the heat pump system. The dependence of the heat pump coefficient of performance on the water temperature is simulated according to the technical data given by the manufacturer. The heat storage capacities of the structures were found to be similar but only the smaller house has storing floor heating installed. The models were verified with measurements.

## RESULTS

As the ground source heat pumps are not dimensioned to meet the full peak demand, the benefit from the optimal control remained the same although the variable electricity costs were reduced by adding the heat pump. Optimal control also increases the self-consumption of the locally produced power.

### Examples of optimised responses

The electricity price depends on the direction of the power flow and was as shown in Fig. 5 in April 2016, when the optimised control signals were as shown in Fig. 7. Fig. 8 shows the optimised SOC of the BES.

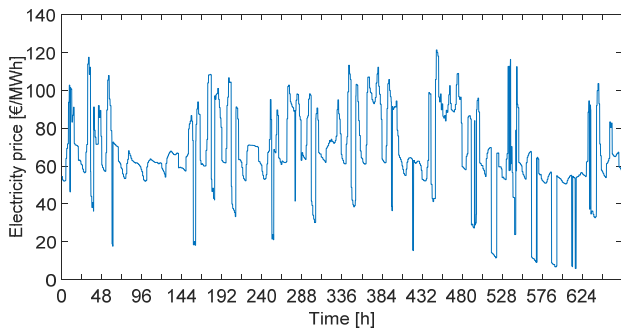


Fig. 5. The electricity price in the optimised solution.

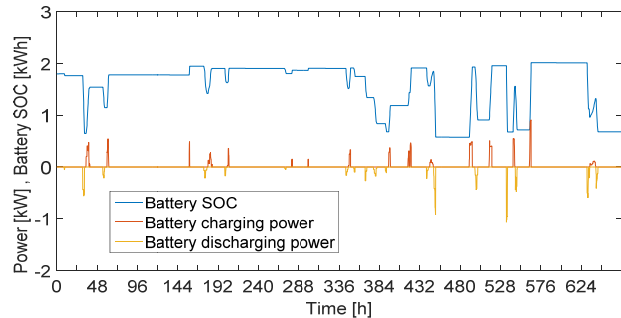


Fig. 6. The battery state of charge (SOC) in the optimised solution in April 2016.

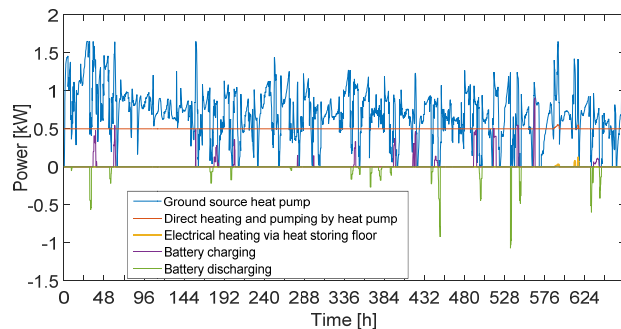


Fig. 7. The controllable electricity loads in April 2016.

### Optimisation benefits from different battery energy storage sizes

In the optimisation results the benefits from the battery energy storage (BES) were very small compared to the benefits from the thermal storage capacity, see Fig. 8. It shows how the extrapolated variable annual electricity costs of the house depend on BES size. The topmost line shows the base case where the house is not controlled based on the electricity price. The lonely spot shows the case when only the BES is optimised and the usage of the thermal storage capacity is as in the base case controlled only based on the temperatures. The red line shows the result when both the BES and the thermal storage capacities are optimally controlled based on the day-ahead forecasts. Then the comfort was not acceptable. The green line shows the case when real time feedback from the indoor conditions that maintains acceptable comfort is added to the day-ahead forecast based optimal control. The magenta line at the bottom shows the optimal control when perfect forecasts are assumed.

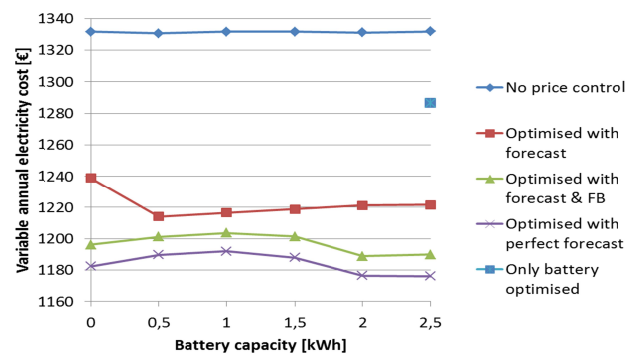


Fig. 8. Benefits of the optimisation with a 5 kW solar panel system, 2.4 kW HP and different BES sizes.

Because of the BESS energy losses the optimisation prefers to utilise the thermal storages whenever possible. In the model the BESS energy losses did not heat the house. From the Fig. 8 we observe that especially with the smaller battery sizes the optimisation method did not converge to the global optimum, because 1) the inverter loss characteristics created multiple optima and 2) the constraints dominate too much because of the relatively small impact of the battery on the total benefit. With the model applied the actual globally optimal solution with the BES is never worse than without the BES. The distance to the assumed global optimum was always small and for many purposes rather insignificant.

The annual benefit when the only price controllable resource was the 2.5 kW BES was about 45 € while the annual benefit from using only the thermal flexibilities for price control was about 136 € and with perfect forecasts about 150 €. The base case annual variable electricity cost was 1332 €

### Optimisation benefits with solar panel sizes

Absolute benefits from optimisation do not change much when the size of solar power is increased. See Fig. 9.

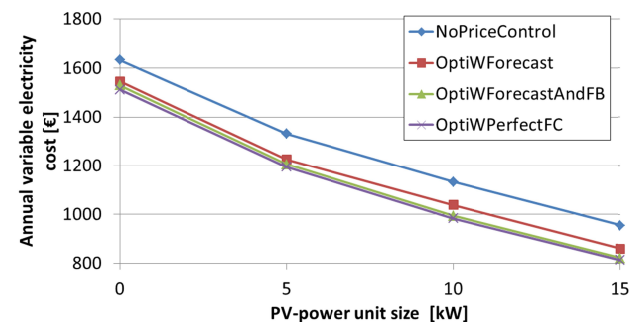


Fig. 9. Benefits of optimisation with the 2kW BES, 2.4 kW heat pump and different solar panel sizes.

### Optimisation benefits with heat pump sizes

The impact of increasing the ground source heat pump size on the optimisation benefits was studied in Fig. 10. Here a 5 kW solar panel is included.



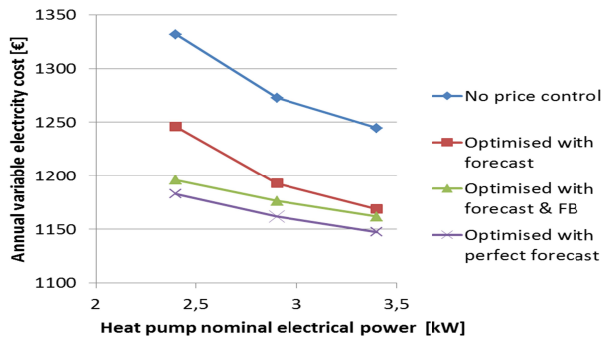


Fig. 10. Benefits of optimisation with different heat pump sizes.

## DISCUSSION

The optimal spot price control applied gives substantial cost savings regardless of the dimensioning of the ground source heat pump and solar panel. The savings are bigger than in our previous paper [3], because the accuracy of weather forecasts has improved. The volatility of spot market prices has not increased at least in these analysed periods. The relatively short time periods 8 weeks and 16 weeks respectively also cause significant stochastic uncertainty to the results.

From the optimisation results it can be seen that a much smaller inverter in the model would have been optimal and thus slightly more favourable for the battery. Inverter dimensioning analysis is left to further research.

The results suggest that it is better to use mainly the thermal load flexibilities and the battery has only a secondary role. When the thermal flexibilities are applied the additional benefits from the battery are so small that the battery is way too expensive to justify the investment. Notice that for reserve market applications the relative advantage of flexible thermal loads is even bigger. The aging of a typical lead acid battery and a LTO battery are crucially different. In these simulations the aging costs of the lead acid battery is almost as big as the benefit from the battery, but the aging cost of the LTO battery is insignificantly small. If the LTO battery is in place due to other reasons such as uninterruptible power supply, demand response can bring some added value to the investment. More detailed studies are needed before final conclusions can be drawn, because here the purpose was only to study to what extent the method applied can be used for this kind of analysis.

Further research plans and suggestions include:

- solving the remaining convergence challenges,
- inverter dimensioning studies,
- grid constraint impact studies and
- detailed comparison of the solutions with LP and MILP approaches.

## CONCLUSIONS

A nonlinear model and method were implemented, applied and developed for the optimal control of the house energy consumption in presence of price signals. The method is based on a constrained nonlinear optimisation method where the gradient is calculated from the adjoint state using the method of Pontryagin. Several initial guesses are generated using heuristics in order to reduce the risk of local optima and other convergence problems. The method has worked very well, when ground source heat pump, storing and direct electrical heating, and solar panel are included as controllable resources in the dynamic energy balance model of the house. When battery energy system was added to the model some problems with local optima and poor convergence were observed especially when the battery was small compared to the other controllable resources. The overall performance of the method is nevertheless already useful for most purposes.

## Acknowledgments

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