

## FUZZY STATE ESTIMATION APPLIED TO SMART DISTRIBUTION NETWORK AUTOMATION FUNCTIONS

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### ABSTRACT

*This paper presents a fuzzy state estimation approach applied to medium voltage distribution networks. The algorithm implemented merge concepts of fuzzy regression, load allocation and robust state estimation. The input measurements uncertainty are modelled using fuzzy sets theory, and the estimated results are fuzzy numbers that represent all possible state values. The results demonstrate several advantages in using proposed methodology as data source for smart distribution network automation functions instead to use the classical state estimation approach.*

### INTRODUCTION

It is well recognized that Distribution System State Estimator (DSSE) is an essential key for implementing smart control strategies such as network restoration, real-time system monitoring, energy loss minimization, outage management, security assessment and Volt/Var optimization [1]. Usually these smart distribution network automation functions are implemented through algorithms based on AC power flow model fed by DSSE output results. To ensure reliability and operational safety, these automation functions must use the rational decision making theory for computing the expected utilities of the consequence of control actions to be applied. To make it possible, it is necessary to quantify the uncertainty related to input information. The problem is that traditional statistical state estimation techniques, generally applied to solving distribution state estimation problem, like Weighted Least Squares (WLS) and Weighted Least Absolute Value (WLAV), not provide any information about the uncertainty of estimated states.

In order to deal with this problem, this paper proposes the use of fuzzy state estimation approach. In this approach, the input measurements uncertainty are modelled using fuzzy sets theory, and the estimated results are fuzzy numbers that represent all possible states values for a given fuzzy measurement set. A fuzzy estimated state ranges from a minimum to a maximal possible value. The difference between these values is called fuzzy state spread and represents the uncertainty related to the estimated state. Smart distribution control functions algorithms can easily use spreads to quantify uncertainties and take more secure and robust actions.

Mathematically, the fuzzy state estimation could be modelled as optimization problem aiming to minimize a

cost function of the spreads of fuzzy measurements. The algorithm implemented is based on a three-phase power system unbalanced model, considering near real-time measurements from different sources and time scales. The complete formulation and algorithm for solving this problem is described. The algorithm merge concepts of fuzzy regression [2], load allocation and robust state estimation. The feasibility of the proposed method is demonstrated with results obtained from a pilot system of CELESC S.A., which is a regional distribution utility of south of Brazil. A comparative analysis, considering distinct measurement scenarios is also performed.

### CLASSICAL STATE ESTIMATION APPROACH

The classical statistical state estimation approach is based on the following measurement model:

$$z = h(x) + e$$

where  $z$  is a  $m \times 1$  measurement vector,  $x$  is a  $n \times 1$  state vector,  $h()$  is a  $m \times 1$  vector of nonlinear functions that relates the measurements  $z$  to the states  $x$ , and  $e$  is a  $m \times 1$  error vector. It is assumed that measurement errors are gaussian and independent.

In a general form, the classical state estimation problem is to determine the state vector  $x$  that minimizes the following cost function:

$$J(x) = \rho(e) = \rho(z - h(x))$$

In a three-phase approach, the equations  $h()$  are basically the active and reactive power flow equations, while the state vector  $x$  contains three-phase complex voltages of all electrical buses.

The cost function  $\rho()$  depends on the estimator technique applied. Weighted Least Squares (WLS) and Weighted Least Absolute Value (WLAV) estimators are widely used [3]. In WLS method  $\rho()$  is the weighted sum of squared residues while in WLAV method is the weighted sum of the residues absolute values. Usually the weights used in both methods is the standard deviation of the gaussian distribution error associated with each measurement.

The WLS and WLAV methods have complementary and distinct characteristics. The WLS method is effective in to filter normal gaussian errors but is sensitive to measures contaminated by gross errors. On the other hand, WLAV

is robust to gross errors but ineffective in filtering gaussian errors.

In order to mix the main qualities of the WLS and WLAV methods developing a state estimator that is effective in treating both gaussian and gross errors several hybrid robust state estimators based on non-quadratic criteria, like Quadratic-Constant (QC), Quadratic-Linear (QL), Square Root (SR) Schweppe-Huber Generalized-M (SHGM) have been proposed [3][4]. These estimators are solved using an iteratively reweighted least squares (IRLS) method [5]. In this method, during the iterative process of solution, penalties are applied to measures with high residues in order to reduce the effect of bad data on the estimated states.

In order to estimate all states the system need to be observable. A power system is fully observable when an available measurement set is enough for compute all power system states. Observability is determined by the type and location of the available measurements as well as by the topology of the network [3]. For a distribution network to be observable is necessary that at least a number of available measurements equal to the number of states to be estimated. However, in a real MV distribution system, just a limited number of real-time measurements are available. To maintain the observability several authors propose the use of active and reactive power pseudo-measurements in order to represent distribution loads.

The reference [6] proposes two approaches for modelling pseudo-measurements; the first is based on correlation and second is based on load probability density functions. The reference [7] proposes the creation of pseudo-measurements through artificial neural networks (ANNs). The use of pseudo-measurements has some limitations since violate some assumptions adopted in the basic formulation of the statistical state estimation problem. In a classic formulation, it is assumed that the errors associated whit measurements are uncorrelated distributed by a gaussian distribution. However, it is known that statistical distribution of electric load variations does not follow any common probability distribution function [8].

Another drawback for using a classical state estimation approach to solve DSSE problem is that, regardless of the method used, no information of uncertainties associated with the estimated states is externalized. This point can be a severe limiting factor for using DSSE function as datasource of control function.

Aiming to smooth these limitations this paper proposes the development of a state estimator for medium voltage distribution networks based on possibility theory (fuzzy) instead of the probabilistic theory.

In the probability theory, uncertainty is represented by a probability distribution constructed from statistical analysis of a population, requiring prior knowledge of the statistical behaviour of the variables under consideration. In the possibility theory uncertainty is modelled by a possibility distribution allowing representing uncertainty even if the statistical characteristics of the variables under consideration are unknown.

## FUZZY STATE ESTIMATION APROACH

### Fuzzy Numbers

A fuzzy number is a generalization of a real number, which does not refer to a single value, but a range of possible values that can be represented by a convex fuzzy set. The fuzzy set theory was developed as an extension of classical set theory (*crisp* sets). In a classical set theory, the membership of elements in a set is a rigid rule with only two possibilities: belonging or not belonging to the set. On the other hand, in the fuzzy set theory, each element has a degree of membership associated with each set, so that an item can belong to more than one set at same time with distinct degree of membership.

Fuzzy numbers has been widely used for modelling vague or inaccurate information, such as linguistic variables like: high, hot, fast, strong etc... In this paper we used triangular fuzzy numbers to represent unknown or uncertain measure values in distribution power system.

A triangular fuzzy number  $A$  can be defined by three points  $A = (a_L, a_C, a_R)$  and a membership function,

$$\mu_A(x) = \begin{cases} 0, & x < a_L \\ \frac{x-a_L}{a_C-a_L}, & a_L \leq x \leq a_C \\ \frac{a_R-x}{a_R-a_C}, & a_C \leq x \leq a_R \\ 0, & x > a_R \end{cases} \quad (1)$$

where  $a_L, a_C, a_R$  is respectively the left, center and right value. Figure 1 illustrates a general triangular fuzzy number.

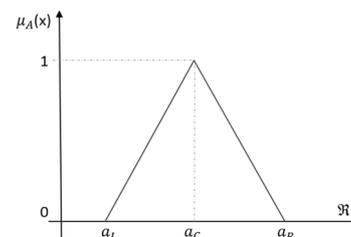


Figure 1- Illustrative example of a triangular fuzzy number

### Measurement and Load Modelling

In our fuzzy state estimation approach, all measurement and loads are represented as a triangular fuzzy measurement where the left value represents a minimal possible value, the centre value represents the most credible value and the right value represents a maximal possible value.

### Real-Time and Near Real-Time Measurements

A typical MV distribution network normally have only a limited number of real-time measurements available in SCADA system. Generally are available three-phase voltage, power flows or currents in a feeder root node and in a few dispersed devices like reclosers and automatic voltage regulators. Also can be available to state estimator,

measurements from distribution transformer and secondary consumers gathered from Automatic Meter Reading (AMR) system.

In several cases, the quality of measurements available is poor due to different time scales and latency of the employed data transmission technology. In some cases, there are measurements just in one phase.

For dealing with these issues, the fuzzy measurements originated from real-time and near real-time measurements are computed by the following equation:

$$FuzMeas = (rtv - \alpha \cdot rtv, rtv, rtv + \alpha \cdot rtv) \quad (2)$$

where *rtv* is a real-time measurement value and  $\alpha$  is a constant coefficient set for each type and source of measurement. The value of coefficient  $\alpha$  is also adjusted in case of delayed measures values.

**Load Modelling**

In a real MV distribution system, there are lot of unmonitored loads. To ensure network observability it is necessary treat and represent these loads in the state estimation problem through active and reactive power injections. These power injections are modelled as fuzzy measurements considering typical fuzzy load shapes and billing data. The typical fuzzy load shapes are constructed from field measurements campaigns.

**Fuzzy State Estimation Algorithm**

The main objective of the fuzzy state estimation algorithm is to determine all state variable values from a given fuzzy measurement set and an electrical network model. The main requirement is that states should be represented in triangular fuzzy form. The left value of triangular fuzzy state should be a minimal possible value, the center value should be the most credible value and the right value should be a maximal possible value. Figure 2 shows a simplified flow diagram of the proposed fuzzy state estimation algorithm. As illustrated, the algorithm is based in two main procedures: a load allocation procedure and a crisp state estimation procedure.

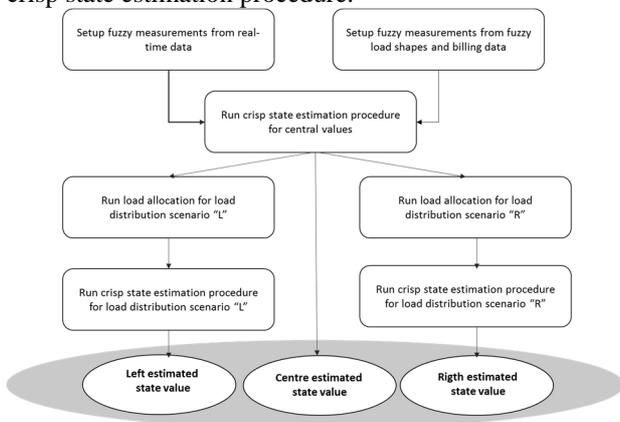


Figure 2 - Flow diagram of the proposed fuzzy state estimation algorithm

**Load Allocation Procedure**

The load allocation procedure is used to build up two load distribution scenarios that will be used by crisp state estimation procedure to determine the left and right value of the estimated states. The load distribution scenario *L* is built in order to have minimal possible voltage values, while the distribution scenario *R* is built in order to have maximal possible voltage values.

The load allocation is performed based on the concept of constrained zone. A constrained zone may be defined as a region of a distribution system bounded by series devices with available power flow measurements. To illustrate the concept of constrained Figure 3 shows a hypothetical MV distribution system with 3 constrained zones.

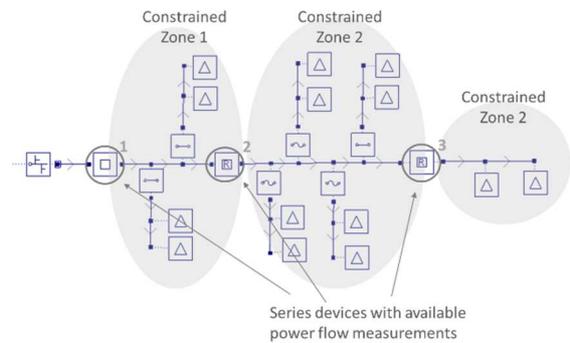


Figure 3 - Illustrative example of constrained zoned concept

The load distribution scenarios are determined through a heuristic procedure. The first step is to compute the measured power of each constrained zone summing power flow measurements of interface devices. After that, for each constrained zone, the loads are sorted in ascending or descending order by the central value of the estimated voltage. Then, the measured power value is distributed between loads respecting the maximum and minimum possible values.

**Crisp State Estimation Procedure**

The crisp state estimator used in our approach, is based on WLS classical estimator with a basic difference: instead to use standard deviation as a weight factor we use the spread of the respective fuzzy measurement. So, the state estimation problem is to solve the following objective function:

$$\min [z - h(x)]^t W^{-1} [z - h(x)] \quad (3)$$

where *W* is a diagonal weighting matrix, with diagonal elements  $w_{ii} = 1/\psi_i$ , and  $\psi_i$  is the *i*th fuzzy measurement spread.

The equation (3) is solved using the Newton-Raphson's method. Due to the characteristics of sparsity and ill-conditioning present in the most distribution systems, the algorithm implemented uses factorization techniques based in Givens rotation using two or three multipliers, oriented by line or column, which ensures the desired numerical robustness.

## EVALUATION AND TESTS

The proposed fuzzy state estimator approach was evaluated using a real distribution system considering four different measurement scenarios:

- **Scenario 1:**
  - Real-Time measurements: PQV measurements located at feeders circuit breaker and reclosers;
  - AMR measurements: unavailable;
  - Normal Error: applied to central value of PQ fuzzy measurements located at distribution loads;
  - Bad data: not applied;
- **Scenario 2:**
  - Real-Time measurements: PQV measurements located at feeders circuit breaker and reclosers;
  - AMR measurements: unavailable;
  - Normal Error: applied to central value of PQ fuzzy measurements located at distribution loads;
  - Bad data: a 20% constant error applied to central value of PQ fuzzy measurements located at distribution loads;
- **Scenario 3:**
  - Real-Time measurements: PQV measurements located at feeders circuit breaker and reclosers;
  - AMR measurements: PQV measurements located at AMR consumer distribution transformer;
  - Normal Error: applied to central value of PQ fuzzy measurements located at distribution loads;
  - Bad data: not applied;
- **Scenario 4:**
  - Real-Time measurements: PQV measurements located at feeders circuit breaker and reclosers;
  - AMR measurements: PQV measurements located at AMR consumer distribution transformer;
  - Normal Error: applied to central value of PQ fuzzy measurements located at distribution loads;
  - Bad data: a 20% constant error applied to central value of PQ fuzzy measurements located at distribution loads;

The fuzzy state estimation was performed for each scenario considering the value of AMR and real-time measurements computed from a power flow analysis that takes as input the central value of PQ fuzzy measurements located at distribution loads.

The magnitude of estimated fuzzy voltages was evaluated using the following measures: Mean Absolute Percentage Error (MAPE), Mean Absolute Percentage Spread (MAPS), Maximum Percentage Error (MAXPE) and Maximum Percentage Spread (MAXPS). These measures are defined as follow:

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{V_C^e - V^r}{V_C^e} \right|$$

$$MAPS = \frac{1}{n} \sum_{i=1}^n \left| \frac{V_L^e - V_R^e}{V_C^e} \right|$$

where  $V^e = (V_L^e, V_C^e, V_R^e)$  is the estimated fuzzy voltage and  $V^r$  is the simulated voltage obtained from power flow analysis.

### Distribution Network

The distribution network used in evaluation test procedures is compound by three radial MV (13,8 kV) urban/rural feeders with more than 242 km and 500 distribution transformers. The main characteristics of this network is presented in table I and a georeferenced diagram is showed in Fig.4.

Table I – Main characteristics of distribution network

Feeders	DTs	Nodes	Reclosers	AMR Consumers	Length (Km)
Feeder 1	226	1586	-	9	89,27
Feeder 2	283	2147	3	7	128,67
Feeder 3	72	488	-	18	24,50
<b>Total</b>	<b>581</b>	<b>4221</b>	<b>3</b>	<b>34</b>	<b>242,44</b>

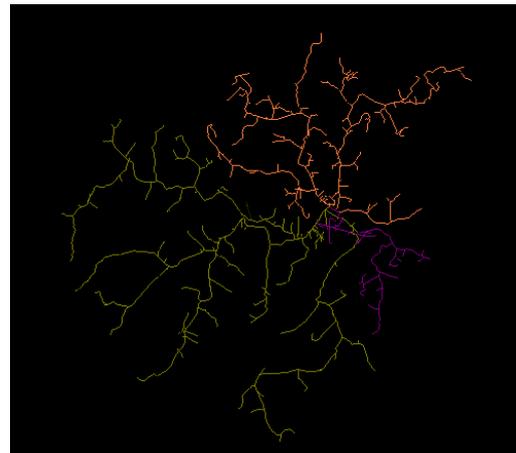


Figure 4 – Georeferenced diagram of distribution network

### Results

Figure 5 shows a MAXPS and MAPS comparison for the four considered scenarios. The MAXPS and MAPS are directly related to fuzzy spreads and reflect the level of uncertainty associated with the estimated states. The results shows clearly that the addition of AMR measurements contribute significantly to reducing the level of uncertainty. It is also observed, that the level of uncertainty not changed significantly due to the addition of bad data.

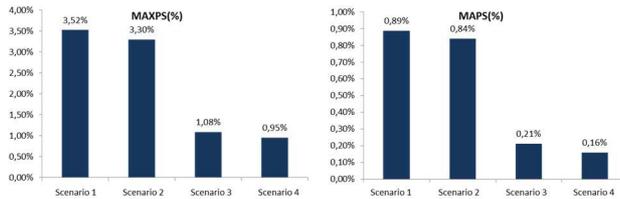


Figure 5 –MAXPS e MAPS comparison for the four considered scenarios

Figure 6 shows a MAXPE e MAPE comparison for the four considered scenarios. It shows that accuracy of centre value estimation is very affected in presence of Bad Data (scenario 2). In scenario 4, where in addition to real-time measurements were considered AMR measurements, this impact was softened.

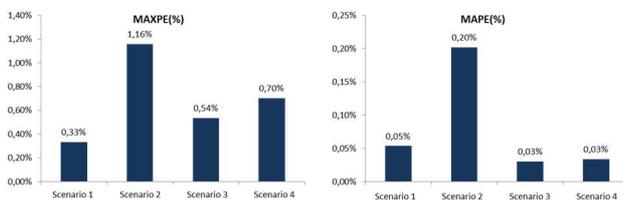


Figure 6 –MAXPE e MAPE comparison for the four considered scenarios

## CONCLUSION

This paper presents a fuzzy state estimation approach applied to medium voltage distribution networks. In this approach, the input measurements uncertainty are modelled using fuzzy sets theory, and the estimated results are fuzzy numbers that represent all possible states value for a given fuzzy measurement set. The validation and tests results demonstrate the feasibility of the proposed method. It is easy to realise that smart distribution control functions algorithms, fed by the fuzzy state estimation results, can easily quantify uncertainties from fuzzy state spreads, and take more secure and robust control actions based on rational decision-making theory.

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