REQUIREMENTS OF STATE ESTIMATION IN SMART DISTRIBUTION GRID

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ABSTRACT

Advanced analytical applications for global controls and optimization of distribution systems have not been widely adopted by utilities because of lacking real time complete system models. As more Intelligent Electronic Devices (IED) with two-way communications are being employed, the amount of quasi real time data gathered at different rates by various automation systems is increasing. With effective integration of data from IED, smart meters, feeder and substation automation systems, a series of steady state snapshots distribution state estimations (DSE) will be a key feature to enable real-time optimization, adaptive protection and control, pricing signal, demand response, and many other smart grid features. This paper briefly reviews the state of the art of DSE, focusing on the requirements for smart distribution grid applications, the effects of distribution network characteristics and bad data detection capability.

INTRODUCTION

Utilities have faced many changes in distribution network. There are more distributed generators (DG), Plug-in Electric Vehicle (PEV), IED, and phasor measurement unit (PMU) in distribution system. The introduction of DG is in accordance with one of the smart grid visions presented by European Commission Community Research, to provide connection access to all network users, particularly for renewable power sources and high efficiency local generation with zero or low carbon emissions [1]. Distribution system states have become more dynamic due to the integrations of many intermittent distributed generation and loads. Due to these changes, utility companies have to control and protect the bi-directional power flow network, to improve power quality and may have to defer investment because of environmental concerns. In order to deal with these issues, a near real time distribution network model becomes necessary for more resilient control. SCADA systems have been installed in distribution network to monitor bus voltage, branch power flow, and other parameters at certain nodes in the distribution feeders. Economic constraint makes it impossible to install the measurement devices in every place where it is needed. Additionally, measurement data are subject to error due to the nature of measurement device and communication problem. Recent widespread deployment of advanced metering infrastructure (AMI) and sensors in the distribution network provides the utilities with a better insight about network conditions as well as its customer behaviour [2]. With adequate integrations of heterogeneous data, DSE could provide a solution to further increase the system model accuracy. DSE is the first step to efficient system operations. Bad data detection is important for effective state estimation (SE), which detects the existence of gross errors in the measurement data, identifies and eliminates them. DSE enables real-time optimization, adaptive protection and control, pricing signal, demand response, and many other smart grid features. DSE results provide more accurate information for spatial load forecast that can be used in network planning. Many efforts have been made to design DSE that is comparable to well established SE techniques used in the transmission system control centre. However, SE methods used in transmission system are not directly applicable to the distribution network due to different characteristics of the two systems. The distribution system has the following characteristics:

1) High r/x ratio
2) Three phase unbalance system
3) Radial topology
4) Limited amount of real time measurements

In order to assist effective system operation, DSE needs to deliver accurate distribution model and handles the aforementioned distribution grid characteristics efficiently. Most of the proposed DSE algorithms are based on weighted least square (WLS) approaches that differ mainly in the choice of state variables and measurements used. Bus voltage magnitude and phase angle (polar form) were used as state variables in [3] while bus injection current measurement based formulation with bus voltages expressed in rectangular form was adopted in [4]. Baran and Kelley introduced a branch current based DSE algorithm in [5] that uses branch current in rectangular form as state variables. This method further refined in [6] by Wang and Schulz.

Some alternative techniques have been presented as well. Ghosh et al. [7] proposed a probabilistic approach that treats the real measurements as solution constraints. A load estimator algorithm that simplifies the network into several measurement areas was proposed in [8]. In [9], Naka et al. proposed a DSE algorithm based on particle swarm optimization.

This paper presents a comparative investigation of DSE between branch current and bus voltage based WLS methods. Comparison of required DSE features such as solution accuracy, computation time, and bad data detection capability in different network topologies are investigated as well as the effects of the inclusion of voltage measurements in the estimation.

DISTRIBUTION STATE ESTIMATOR

The relation between a measurement \( z \) and the state variables \( x \) can be expressed as follow:

\[
z = h(x) + r
\]
Where \( h(x) \) is the function relating the measurement to the state variable vector \( x \). \( r \) is the measurement residual. The estimated system state is obtained by minimizing the following objective function:

\[
\min_x f(x) = \sum_{i=1}^{m} W_i (z_i - \hat{h}_i(x))^2
\]

\[
= [z - h(x)]^T W [z - h(x)]
\]  

(2)

\( W \) is the measurement weight matrix with \( W_{ii} = \frac{1}{\sigma_i^2} \). \( \sigma_i \) is the standard deviation of measurement \( i \). The following normal equation is solved iteratively to compute the state variable updates and \( x^{k+1} = x^k + \Delta x^k \).

\[
[G(x^k)] \cdot \Delta x^k = H^T(x^k) \cdot W \cdot [z - h(x^k)]
\]  

(3)

Where \( G(x^k) = H^T(x^k) \cdot W \cdot H(x^k) \). \( H(x^k) \) is the measurement Jacobian matrix. To include zero injection measurements, equality constraints can be included by adding a plenty term to (2). Two existing DSE algorithms compared in this study are described in the followings.

**Bus Voltage Based DSE**

In [3], a state estimation method using three phase feeder model with bus voltage magnitude and phase angle as state variables was proposed. The method is based on the WLS approach and can handle power, voltage, and current measurements. Another WLS state estimator using bus voltage in rectangular form as state variables was proposed in [4]. The rectangular bus voltage algorithm uses a bus injection current measurement formulation in which the power measurements are converted to their equivalent current measurements. The entries of measurement Jacobian matrix become constant and equal to the admittance matrix elements. Voltage magnitude measurements are converted to their equivalent rectangular form using phase angle information obtained from the calculated voltage value, such that the corresponding measurement Jacobian terms are either zero or unity. The following equations are used to convert the power measurements into their equivalent current measurements.

**Branch power flow:**

\[
I_{km}^{\text{mea-eqv}} = \left( \frac{P + jQ}{V_{\text{cal}}^k} \right)^* = \text{Re}(I_{km}^{\text{mea-eqv}}) + j \text{Im}(I_{km}^{\text{mea-eqv}})
\]  

(4)

Where \( I_{km}^{\text{mea-eqv}} \) is the equivalent branch current measurement from bus \( k \) to bus \( m \). \( (P + jQ)_{km}^{\text{mea}} \) is the power flow measurement at branch \( km \). \( V_{\text{cal}}^k \) is the estimated bus voltage at bus \( k \).

**Bus power injection:**

\[
I_{k}^{\text{mea-eqv}} = \left( \frac{P + jQ}{V_{k}^{\text{cal}}} \right)^* = \text{Re}(I_{k}^{\text{mea-eqv}}) + j \text{Im}(I_{k}^{\text{mea-eqv}})
\]  

(5)

\( I_{k}^{\text{mea-eqv}} \) is the equivalent bus injection current measurement of actual power injection measurement at bus \( k \). \( (P + jQ)_{k}^{\text{mea}} \).

**Voltage magnitude:**

\[
V_{k}^{\text{mea-eqv}} = \left| V_{k}^{\text{cal}} \right| = \text{Re}(V_{k}^{\text{mea-eqv}}) + j \text{Im}(V_{k}^{\text{mea-eqv}})
\]  

(6)

\( V_{k}^{\text{mea-eqv}} \) is equivalent bus voltage measurement.

**Branch Current Based DSE**

In [5], a popular branch current based DSE method for radial and weakly meshed distribution feeders, using rectangular form branch current as state variables was introduced. Equations (4) and (5) are used to convert the power measurements into equivalent current measurements which are calculated at each iteration allowing the corresponding power measurement’s Jacobian matrix entries to be either unity or zero. The current based algorithm was improved further in [11] that treats the current measurements more efficiently and nonlinear Jacobian terms are avoided. After being neglected in the early development of current based DSE, voltage measurement was incorporated to the algorithm in [12] and [13]. Wang and Schulz [6] proposed a revised current based SE, using the magnitude and phase angle of branch current as state variables. In addition, meter placement issue was also discussed. It was shown in [6] that among all measurement types, voltage magnitude measurement is the least effective measurement to enhance accuracy of the current based DSE.

**Weight Transformation**

In above mentioned rectangular bus voltage based and branch current based DSE methods, actual measurements are transformed into their equivalent measurements. According to [14], the equivalent measurement variance can be calculated as follows:

\[
s^2 = \sum_{i=1}^{m} \left( \frac{\partial F}{\partial z_i} \right)^2 \sigma_{ii}^2
\]

\[= \left( \frac{\partial F}{\partial z_1} \right)^2 s^2_{11} + \left( \frac{\partial F}{\partial z_2} \right)^2 s^2_{22} + \cdots + \left( \frac{\partial F}{\partial z_m} \right)^2 s^2_{mm}
\]  

(7)

where \( y = F(z_1, z_2, \cdots, z_n) \) is the measurement equivalent function. Assuming the measurements to be independent with each other and applying equation (4)-(6) into equation (7), the equivalent variance are shown in Table 1.
Table 1. Calculation of measurement variances

<table>
<thead>
<tr>
<th>Meas.</th>
<th>Variance ( (R_{\text{eqv}} = \sigma_{\text{eqv}}^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (P + jQ)<em>{\text{mea}} )[\downarrow] ( I</em>{\text{mea-eqv}} )</td>
<td>( \sigma_{\text{eqv-r}}^2 = \frac{e_k^2}{e_k^2 + f_k^2} \sigma_i^2 + \frac{f_k^2}{e_k^2 + f_k^2} \sigma_i^2 )</td>
</tr>
<tr>
<td>( (P + jQ)<em>{\text{mea}} )[\downarrow] ( I</em>{\text{mea-eqv}} )</td>
<td>( \sigma_{\text{eqv-x}}^2 = \frac{f_k^2}{e_k^2 + f_k^2} \sigma_i^2 + \frac{e_k^2}{e_k^2 + f_k^2} \sigma_i^2 )</td>
</tr>
<tr>
<td>(</td>
<td>V</td>
</tr>
<tr>
<td>(</td>
<td>V</td>
</tr>
</tbody>
</table>

Where \( e_k \) and \( f_k \) are the real and imaginary parts of estimated voltage at bus \( k \). For every equivalent measurement, \( \sigma_{\text{eqv-r}}^2 \) and \( \sigma_{\text{eqv-x}}^2 \) are the equivalent variances of its real and imaginary parts.

**CASE STUDY**

The performance of existing polar node voltage based (NVP), rectangular node voltage based (NVR), and rectangular branch current based (BCR) WLS DSE formulations are compared. The performance of DSE methods is evaluated by using Taiwan Power Company (TPC) 3-phase unbalance 39 bus test system. The line parameter is 0.0498 + 0 j 0.1202 Ω/km.

**Figure 1. TPC 39 bus test system**

Three measurement types are included in the tests. Distribution automation systems provide real-time measurements containing bus voltage and feeder branch real and reactive power measurements. Zero injections are considered as virtual measurements and treated as equality constraints. Two types of pseudo-measurements are considered. Pseudo-measurements based on AMI data that include voltage magnitude and bus power injection measurements. In the simulations, measurement’s standard deviation (\( \sigma_i \)) is calculated as follows:

\[
\sigma_i = \frac{e_i}{3 \times 100}
\]

where \( e_i \) is the percentage of measurement error.

**Performance Indices**

In order to compare the accuracy and execution time of different methods, 1,000 samples with random error based on normal distribution are generated for each test case. Root mean square errors (RMSE) are calculated and used for accuracy comparison.

\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{V}_i - V_i)^2}
\]

where \( \hat{V}_i \) is estimated voltage magnitude and phase angle at bus \( i \), \( V_i \) is the true voltage value obtained from load flow solution. \( n \) is the number of buses.

**Test Results**

Test 1 aims to measure the level of accuracy and execution time of different DSE methods. In addition to the performance comparison, effectiveness of AMI based power injection and voltage magnitude pseudo-measurements are also investigated. It is assumed that bus power injection data based on customer meter readings are available every 15 minutes and therefore have less error compared to those calculated based on monthly billing data in conjunction with typical load profiles. Voltage magnitude measurements are also available from the AMI readings. Table 2 shows the measurement types, locations and standard deviations of different test cases.

**Table 2. Test Cases in Test 1**

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Meas. Type</th>
<th>Meas. Location</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( PQ_{\text{flow}} ) (FTU)</td>
<td>1-2, 3-4</td>
<td>0.017</td>
</tr>
<tr>
<td>2</td>
<td>( PQ_{\text{inj}} ) (Pseudo meas.)</td>
<td>all load buses</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>( PQ_{\text{flow}} ) (FTU)</td>
<td>1-2, 3-4</td>
<td>0.017</td>
</tr>
<tr>
<td>4</td>
<td>( PQ_{\text{inj}} ) (AMI)</td>
<td>all load buses</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>(</td>
<td>V</td>
<td>) (AMI)</td>
</tr>
</tbody>
</table>

Figures 2 to 4 show the RMSE and execution times of tested methods. Among them NVP and NVR based methods have higher solution accuracy than BCR based method, particularly on the estimated voltage magnitude. On the other hand, BCR and NVR based methods outperform NVP based method in the execution time. From Figures 2 and 3, it is also evident that the accuracy has increased with the introduction of power injection and voltage magnitude from AMI.
Test 2 investigates the effect of using variance transformation for the measurements used in NVR and BCR based methods. Test case 1 uses the original measurement variance as the weight in the calculation. While test case 4 uses variance transformation formula presented in Table 1 to find the measurement weights.

Fig. 5 shows the accuracy comparison with and without variance transformation in measurement conversion. It can be seen that variance recalculation is required in order to improve the station accuracy. The cost to the increase of estimation accuracy is highlighted in Fig. 6. The execution time is increased due to the recalculation of weight matrix and the gain matrix in each iteration. In this study, weight matrix is recalculated only in the first and second iteration. Average numbers of iterations required in the NVR and BCR based methods to reach the final solution are 2 and 3 respectively.

Test 3 investigates the difference in bad data detection for radial and weakly meshed feeder topologies. For this purpose, a loop has been introduced in the test feeder by adding a new 2.7 km branch between bus 11 and bus 32 of

Table 3. Test Cases in Test 2

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Meas. Type</th>
<th>Meas. Location</th>
<th>Weight Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( P_{Q_{flow}} ) (FTU)</td>
<td>1-2, 3-4</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>( P_{Q_{inj}} ) (Pseudo meas.)</td>
<td>all load buses</td>
<td>( \frac{1}{\sigma_{ii}^2} )</td>
</tr>
</tbody>
</table>

Variance Transformation (Table 1)
the TPC test system shown in Fig. 1. Bad data was introduced to measurement \( PQ_b \) by replacing it with 300% of its actual value. NVR based method with weight transformation is used in this test.

Weakly meshed network (Test case 6) has higher measurement redundancy as compared to the radial network (Test case 5). Table 4 shows that the NVR based DSE can correctly locate the bad measurement data in the weakly meshed network but it failed to do so in radial network, despite the relatively high error used in this test. Bad data detection is based on normalized residuals. The result suggests that bad data detection is one of the crucial challenges in the DSE that needs to be addressed since most of the distribution feeders are radial.

**Table 4. Bad Data Detection Test**

<table>
<thead>
<tr>
<th>Meas. ( PQ_{\text{flow}} ) (( \sigma=0.01 ))</th>
<th>Case 5</th>
<th>Case 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( PQ_{\text{inj}} ) (( \sigma=0.05 ))</td>
<td>All load buses</td>
<td></td>
</tr>
</tbody>
</table>

**Feeder Topology**

| Radial | Weakly meshed |

**Bad Data**

| \( PQ_b \) (300% error) |

**Largest \( r^N \) Location**

| \( PQ_{11}^b \) | \( PQ_{6}^b \) |

**CONCLUDING REMARKS**

This paper presents a comparison between three existing WLS based DSE formulations, namely NVP, NVR, and BCR based methods. Based on the results obtained from 1000 simulations in each test case, it is found that NVP and NVR based methods deliver better solutions. However, the average execution time of NVP is higher than those of NVR and BCR based methods, making it less attractive for distribution network applications since distribution network has much higher number of busses. The use of AMI data in DSE provides a more accurate solution which is not only due to the lower bus power injection measurements error, but also the inclusion of voltage magnitude measurements. Test results also suggest that the application of measurement variance transformation in NVR and BCR based methods would improve the solution accuracy, although it requires additional computation and results in a longer execution time. Bad data detection is one of the important issues in DSE. More robust technique needs to be developed to deal with bad measurement data in both radial and weakly meshed distribution networks.

**REFERENCES**


