OPTIMAL LOCATION OF MEASUREMENT DEVICES IN DISTRIBUTION GRIDS VIA BOOLEAN CONVEX OPTIMIZATION

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ABSTRACT
This paper presents a boolean convex optimization method to identify optimal locations for placing additional measurement devices to improve the standard deviation of estimated voltages across a distribution system with DG. The meter placement problem is formulated as an optimal experimental design task. The benefits of the proposed technique in terms of reduced estimation errors are revealed by numerical simulations on a 55-bus distribution network.

INTRODUCTION
Deregulation has forced the electric utility industry to optimize the power system operational efficiency, leading to less investment in new facilities and pushing the system to be exploited closer to its limits. In this environment, the operation philosophy of distribution grids has changed, making them more active and creating modern monitoring and control tasks. These tasks are accomplished by developing suitable and reliable monitoring systems, where the design of the measurement infrastructure is a critical element for their effective operation. A key issue in future distribution control will be state estimation (SE), which provides effective operation. A key issue in future distribution control will be state estimation (SE), which provides

\begin{align*}
\text{PROBLEM FORMULATION} \\
\text{The state estimation measurement model is given by} [1]: \\
\quad z = h(x) + e \quad (1)
\end{align*}

where \( z \in \mathbb{R}^m \) is the measurement vector, \( h(x) \in \mathbb{R}^{2n-1} \rightarrow \mathbb{R}^m \) is a vector-valued nonlinear function of the states, \( x \in \mathbb{R}^{2n-1} \) is the true state vector consisting of the \( n-1 \) phase angles and the \( n \) magnitudes of nodal voltages, \( e \in \mathbb{R}^m \) is the measurement noise vector (Gaussian random variable with \( E(e) = 0 \) and \( E(e e^T) = R = \text{diag}(\sigma_i^2) \)), where \( \sigma_i^2 \) is the variance of the \( ith \) measurement error), and \( n \) is the number of buses.
Assuming that the distribution grid is observable with the existing measurements, the maximum likelihood estimate $\hat{x}$ of $x$ is computed by the following iterative procedure:
\[
G(x^k)(x^{k+1} - x^k) = H^T(x^k)R^{-1}(z - h(x^k))
\]
where, $k$ is the iteration index, $H(x^k) = \frac{\partial h(x)}{\partial x}$ is the $m \times (2n-1)$ Jacobian matrix and $G(x^k) = H^T(x^k)R^{-1}H(x^k)$ is the $(2n-1) \times (2n-1)$ gain matrix.

Let $H(\hat{x})$ be the Jacobian matrix of existing measurements evaluated at prior estimate $x = \hat{x}$ obtained by (2). The problem of optimal meter placement is to choose a subset of $m_s$ measurements, from a set of $m_c$ candidate measurements, to minimize a scalar-valued function of the state error covariance matrix:
\[
C(\hat{x}) = G^{-1}(\hat{x}) = \left( H^T(\hat{x})R^{-1}H(\hat{x}) \right)^{-1}
\]
where the $i$th diagonal entry $C_{ii}$ of $C$ is the variance of the $i$th state.

Assuming that $h_{ij}^c(\hat{x})$ is the Jacobian row associated with the $i$th candidate measurement, the gain matrix $G(y) \in \mathbb{R}^{m \times n}$ of existing and candidate measurements is:
\[
G(y) = G(\hat{x}) + \sum_{j=1}^{m_c} y_j \left( \frac{h_{ij}(\hat{x)}h_{ij}^T(\hat{x})}{\sigma_{ij}^2} \right)
\]
where $\sigma_{ij}$ is the standard deviation of the $i$th candidate measurement and $y_j$ is a binary decision variable indicating whether the $i$th meter is to be used.

The optimal measurement location problem is formulated as an optimal experimental design problem [16]:
\[
\begin{align*}
\text{min} & \quad f(C(y)) \\
\text{s.t.} & \quad \mathbf{1}^T y = m_s \\
& \quad y \in \{0,1\}^{m_c}
\end{align*}
\]
where $C(y) = G^{-1}(y)$, $m_s < m_c$, and $\mathbf{1}$ is the vector with all entries one.

Different measures of estimation quality [17] can be used as objective functions $f(C(y))$, as shown in Table 1.

### Table 1 Optimal experimental design formulations.

<table>
<thead>
<tr>
<th>Optimal Design</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$\text{trace}(C(y))$ or $-\text{trace}(G(y))$</td>
</tr>
<tr>
<td>$D$</td>
<td>$\text{logdet}(C(y))$ or $-\text{logdet}(G(y))$</td>
</tr>
<tr>
<td>$M$</td>
<td>$\max_i (C(y)_{ii})$</td>
</tr>
</tbody>
</table>

Formulation (5) is a boolean (binary) convex optimization model [17].

An optimization problem of the form:
\[
\begin{align*}
\text{min} & \quad c^T y \\
\text{s.t.} & \quad A(y) = A_0 + \sum_{i=1}^{r} y_i A_i \geq 0
\end{align*}
\]
where $A_0$ and $A_i$, $i = 1, \ldots, r$ are real symmetric matrices, is called semidefinite programming (SDP) subject to the linear matrix inequality (LMI) constraint, where $A \geq 0$ designates a semi-positive definite matrix, i.e., a symmetric matrix with non-negative eigenvalues. The $A$ and $M$ optimal design problems can be transformed to SDP models [16], [18], as shown in Table 2.

### Table 2 Formulation of $A$ and $M$ optimal design problems as SDP.

<table>
<thead>
<tr>
<th>$A$-optimal design</th>
<th>Objective function</th>
<th>( \mathbf{1}^T y \quad (t \in \mathbb{R}^{m}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMI constraint</td>
<td>( G(y) \begin{bmatrix} I \ I \end{bmatrix} + \sum_{i=1}^{m} y_i \begin{bmatrix} G_i \ 0 \end{bmatrix} + \sum_{i=1}^{r} t_i \begin{bmatrix} 0 \ 0 \ 0 \ e_{nt}^T \end{bmatrix} \geq 0 )</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>$M$-optimal design</th>
<th>Objective function</th>
<th>( t \quad (t \in \mathbb{R}) )</th>
</tr>
</thead>
</table>
| LMI constraint      | \( G(y) \begin{bmatrix} I \\ I \end{bmatrix} \geq 0 \)
|                     | \( G(y) \begin{bmatrix} I \\ 0 \end{bmatrix} + \sum_{i=1}^{m} y_i \begin{bmatrix} G_i \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \end{bmatrix} \geq 0 \) |

### SIMULATION RESULTS

To show the effectiveness of the optimization procedure, the proposed methodology has been applied to a 55-bus distribution network with DG, depicted in Fig. 1, whose data can be found in [5]. The load flow data are stored in an ASCII file with the PTI PSS/E “raw” format [20], whereas the data for measurement types and locations are given in a csv file. The open-source MATLAB [19] based power system simulation package MATPOWER [20] has been modified accordingly to read these files and solve the state estimation problem. The solution of problem formulations shown in Table 1 is derived using MATLAB based optimization modeling tool YALMIP [21], together with an internal branch-and-bound (B&B) framework and the non-commercial lower bound solver SDPT3 [22] for the SDP relaxation at each node of the B&B tree. To solve the SDP problems of Table 2, we also apply YALMIP, using its internal outer approximation implementation, and the solver SCIP [23] for the resulting Boolean integer LP (BILP) problems. MATLAB code has been also developed to write to and read from ASCII files, the LMI data in sparse SDPA format [24].
State variances, i.e. diagonal elements of matrix $G(v)$, are checked before and after the placement of eligible measurements. Four optimal design formulations are used to find optimum measurement locations. The existing measurement set consists of 10 voltage magnitudes at slack bus and DG buses, 6 pairs of P/Q power flows through branches connected to bus 71 and pairs of P/Q injections at all buses. The eligible measurement set comprises 21 pairs of P/Q power flows at the top of the 3 feeders and voltage magnitudes at 16 randomly selected load buses ($m_s = 58$). In order to run state estimation, all measurement values are obtained by power flow solution. Each measurement type is assigned a distinctive variance $\sigma^2$: $16 \times 10^{-6}$ for power flow, $4 \times 10^{-6}$ for power injections, and $25 \times 10^{-8}$ for voltage magnitudes.

Figures 2-7 show the fluctuation of state variances per bus, considering 3 cases for $m_s$: 15, 30, and 45. Each figure illustrates 5 curves: one related to the existing measurement set and four associated with the optimal design placements, that is, A-optimal trace, D-optimal logdet, and A-design SDP, and M-optimal SDP. Referring to Fig. 2, the improvement in voltage angle variances is significant regarding A-optimal design using logdet measure. A- and M-optimal SDP formulations, along with trace measure, result in considerably lower reduction in voltage angle variances. On the contrary, regarding Fig. 3, SDP formulations reduce radically voltage magnitude variances, while placement using logdet and trace measures lead to rather lower reductions. Figures 4-5 demonstrate approximately the same behavior regarding all 4 methods in case of $m_s = 30$.

As Fig. 4 shows, the increase on measurements to be placed, leads to further improvement in voltage angle variances concerning trace measure and SDP formulations. From Fig. 5 is obvious that voltage magnitude variances approach zero in case of SDP formulations and are appreciably decreased in case of logdet and trace formulations, as well. Fig. 6-7 show further improvement in state variances as candidate measurements for location increase ($m_s = 45$). Although voltage magnitude variances are minimized, voltage angle variances are characterized by considerably lower improvement for all formulations.
As expected, placement of measurements improves all state variances and as their number rises better results are obtained. The main variance peaks are located in the regions close to load buses, e.g. buses 14, 18 and 20, and are more resistant to reduction. The locations of the selected measurements for each experimental design formulation are shown in Tables 3-5. Table 3 shows that solution of A-optimal SDP coincides with that of M-optimal SDP. Voltage magnitude and reactive power flow measurements are preferable. On the contrary, active and reactive power flow measurements are primarily eligible in case of A-design using logdet and trace measure.

| Table 3 Number and optimal locations of selected measurements for A- and M-optimal design with SDP. |
|---|---|
| \( m_{s} \) | Location |
| 15 | \( V_1, V_2, V_4, V_5, V_6, V_7, V_12, V_13, V_15, V_17, V_{19}, V_{20} \) |
| 30 | \( V_1, V_2, V_4, V_5, V_6, V_7, V_12, V_13, V_14, V_15, V_{17}, V_{18}, V_{19}, V_{20}, Q_{37-38}, Q_{38-39}, Q_{39-40}, Q_{41-42}, Q_{42-43}, Q_{43-44}, Q_{44-45}, Q_{45-46}, Q_{46-49}, Q_{49-50}, Q_{49-56}, Q_{60-61}, Q_{81-80} \) |

| Table 4 Number and optimal locations of selected measurements for the A-optimal design (trace). |
|---|---|
| \( m_{s} \) | Location |
| 15 | \( P_{31-32}, P_{33-34}, P_{36-37}, P_{37-38}, P_{38-39}, P_{39-40}, P_{40-41}, Q_{39-40}, Q_{60-61} \) |
| 30 | \( V_1, V_2, V_5, V_6, V_7, V_9, V_{12}, V_{14}, V_{15}, V_{20}, P_{32-31} \) |

| Table 5 Number and optimal locations of selected measurements for D-optimal design (logdet). |
|---|---|
| \( m_{s} \) | Location |
| 15 | \( P_{31-32}, P_{33-34}, P_{36-37}, P_{37-38}, P_{38-39}, P_{39-40}, P_{40-41}, Q_{39-40}, Q_{60-61} \) |
| 30 | \( V_1, V_2, V_5, V_6, V_7, V_9, V_{12}, V_{14}, V_{15}, V_{20}, P_{32-31} \) |
CONCLUSION
This paper describes a method for placement of new measurements in distribution networks embedded with DG, in order to improve the uncertainty of the estimated bus voltages at every node. The problem is posed under the framework of optimal experimental design, by minimizing scalar-valued functions of the state error covariance matrix. The advantage of the method is that it finds optimal meter locations in simple and computationally efficient manner. Test results using a 55-bus distribution network verify the performance of the proposed model.

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REFERENCES