APPLICATION AND ANALYSIS OF SYNCHROPHASOR-BASED ONLINE IMPEDANCE MEASUREMENT METHODS

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ABSTRACT

Electricity generation from renewable energy sources can be constrained by the static thermal rating of existing overhead lines. A potential solution is dynamic line rating based on the line’s temperature. The conductor temperature can be calculated from the line’s resistance, if measured accurately and in real-time. In this paper three suggested methods for impedance determination from synchronized phasor measurements are compared: the linear, non-linear and total least squares estimators. All methods have previously been put forward for their robustness against noise in the synchrophasor measurements and for accuracy of the identified impedance. The purpose of the comparison is to assess how suitable the methods are for high accuracy applications such as temperature monitoring. The assessment was conducted by applying all three methods to identify impedance values from real synchronized voltage and current measurements. The methods performed similarly, but none was able to identify the impedance with the reliability required for high accuracy applications. The potential reasons for the inaccuracy and future work are discussed.

INTRODUCTION

As part of the transition to a low-carbon economy an increasing amount of electricity is generated from renewable resources. However, connection and generation can be constrained by the static thermal ratings of existing overhead lines (OHL), which are designed to ensure safe operation even during worst case ambient conditions such as high temperature and low wind speed [1]. Despite the fact that these conditions are rarely met, the current must always be below the conservative static limit. A dynamic line rating (DLR) can be established by monitoring the state of the line in real-time, thus enabling increased utilization when ambient conditions have a cooling effect. There are several approaches to obtain a DLR; including monitoring of the changes in sag due to thermal expansion of the line, measuring the tension of the cable or calculating the conductor temperature from weather data recorded close to the line [1].

Alternatively the average temperature of the conductor can be calculated from its electrical resistance. Resistance can be determined using synchronized measurements of voltage and current from the ends of an OHL [2], [3]. These measurements are available from phasor measurement units that are becoming increasingly common at power system substations [4] and do not affect the OHL’s normal operation.

While it is very desirable to increase OHL utilization, this must not compromise safe operation of the power network. Hence, a DLR has to be reliable and report changes in the line’s temperature with a high degree of certainty, which in turn depends directly on the certainty of the determined resistance value. Because the temperature coefficients of conductors are usually small (typical value for copper: 0.004 °C⁻¹), there is a requirement to detect small corresponding changes in resistance (0.04 % for one degree temperature resolution). Therefore a synchrophasor-based method for measuring impedance is only suitable if it identifies the values with a consistently low uncertainty. The method must be robust and insensitive to noise in the utilized voltage and current measurements.

To date various synchrophasor-based impedance measurement methods have been suggested in the literature. The simplest methods apply closed-form calculation using one or two sets of phasor measurements [2], [5]. They perform well if the measurements are perfect and free of noise, but if this is not the case the accuracy of the calculated impedance has been shown to deteriorate [5]. In practice, phasor measurements are always subject to some level of noise introduced in the instrumentation channel and through imperfect synchronization [6]. Systematic errors, for example caused by instrument transformers, can be corrected by introducing calibration constants that are found through long term observation and analysis of the voltage and current measurements [7]. In addition, statistical methods such as the chi-square [3], [8] and Durbin-Watson test [9] can detect and eliminate occurrences of bad data. In order to filter remaining random noise, parameter estimation methods have been proposed.
With this objective the linear Least Squares (LS) estimator was applied in [3], [5], [10]; it requires multiple sets of measurements from different operating conditions of the line and under noisy conditions it yields more accurate parameter values than closed-form calculations. Collection of multiple measurements over a period of time can be problematic for real-time monitoring and the reported uncertainties may not satisfy high-accuracy requirements. In an attempt to resolve these issues non-linear LS estimation has been employed [8], [11], estimating both states and parameters simultaneously in an iterative process and using as little as one measurement set. Another approach is the total LS method [9], [12]; similarly to the non-linear LS estimator it assumes potential noise in all of the voltage and current phasors, but also in the system model itself. In [9] it outperforms the linear LS method even if there is no variation in loading conditions. The Extended Kalman Filter (EKF) has also been successfully applied to the real-time parameter estimation problem [7]. Measurements of ambient temperature as well as wind speed were utilized in the EKF, which may not be available depending on the relevant system. All of these methods have demonstrated strong performance in different specific situations, some of them in simulated, others in real power networks. One remaining question is to understand how they perform relative to each other and how to choose the method that will best meet accuracy requirements given the characteristics of a particular impedance measurement problem.

In this paper the results of a comparative study of the linear, non-linear and total LS estimators are presented. They have all been utilized to measure the known value of a lumped impedance built into an OHL in the Swedish grid using steady-state synchronized voltage and current measurements gathered in a real experiment.

Firstly, the phasor measurement, system model and application of the three estimation methods are explained. This is followed by an outline of the experimental setup. The results are then presented and analyzed with respect to differences between the methods. The paper concludes with a discussion of the results and planned future work.

**METHODS**

In this section the methods for phasor estimation and parameter estimation are introduced.

**Phasor estimation method**

To estimate complex voltage and current phasors from the sampled signals the Discrete Fourier Transform (DFT) was applied [4]. Thus the fundamental component at the nominal power system frequency (50 Hz) was extracted. The window length for each phasor estimation was set to one cycle, which is the minimum length for the DFT; thereby allowing observation of changes in the phasors’ amplitude and frequency over time.

The lumped impedance and overhead line are assumed to have the same parameters for all three phases. Hence, positive sequence components were computed and used for further analysis.

**Parameter estimation methods**

First of all, the system model as well as the associated variables and parameters used to represent the OHL system are defined; followed by an outline of the parameter identification methods.

An OHL is classically modelled by the pi circuit shown in Figure 1.

![Figure 1. Pi circuit diagram for modelling an OHL.](image)

The two complex model equations are based on Kirchhoff’s laws:

\[ V_s - V_r = (I_s - V_r Y_1)Z \]  
\[ I_s - I_r = V_r Y_1 + V_r Y_2 \]

where

- \( V_s, I_s \) positive sequence voltage and current measured at the sending end of the line,
- \( V_r, I_r \) positive sequence voltage and current measured at the receiving end of the line,
- \( Z \) positive sequence series impedance,
- \( Y_1, Y_2 \) positive sequence shunt admittances.

Rearranging (1) and (2) gives [10]

\[ I_s = (V_s - V_r)/Z + V_r Y_1 \]  
\[ I_r = (V_s - V_r)/Z - V_r Y_2 \]

Let \( Y_0 = 1/Z \), then

\[ I_s = (V_s - V_r)Y_0 + V_r Y_1 \]  
\[ I_r = (V_s - V_r)Y_0 - V_r Y_2 \]

\( Y_0, Y_1 \) and \( Y_2 \) are the parameters to be found. The last step makes the equations linear in the parameters. Separating real and imaginary parts gives four real equations:

\[ Re(I_s) = Re(\Delta V)G - Im(\Delta V)B + Re(V_r)G_1 - Im(V_r)B_1 \]  
\[ Im(I_s) = Im(\Delta V)G + Re(\Delta V)B + Im(V_r)G_1 + Re(V_r)B_1 \]
\[ Re(I_v) = Re(\Delta V)G - Im(\Delta V)B - Re(V_r)G_2 + Im(V_r)B_2 \]  
\[ Im(I_v) = Im(\Delta V)G + Re(\Delta V)B - Re(V_r)G_2 - Re(V_r)B_2 \]

where
\[ Y_0 = G + jB, Y_1 = G_1 + jB_1, Y_2 = G_2 + jB_2, \Delta V = V_v - V_r. \]

Converting to matrix form:
\[ I = VY \]

where
\[ I = [Re(I_v) \ Im(I_v) \ Re(I_r) \ Im(I_r)]^T \]
\[ V = \begin{bmatrix} 
Re(\Delta V) & -Im(\Delta V) & Re(V_r) & -Im(V_r) & 0 & 0 \\
Im(\Delta V) & Re(\Delta V) & Im(V_r) & Re(V_r) & 0 & 0 \\
Re(\Delta V) & -Im(\Delta V) & 0 & 0 & -Re(V_r) & Im(V_r) \\
Im(\Delta V) & Re(\Delta V) & 0 & 0 & -Im(V_r) & -Re(V_r) 
\end{bmatrix} \]
\[ Y = [G \ G_1 \ B_1 \ G_2 \ B_2]^T \]

Each measurement set consists of four phasors \( V_0, I_0, V_r, I_r \) (complex variables). For \( n \) sets the matrices are extended accordingly so that their dimensions become \( I \in \mathbb{R}^{4n} \) and \( V \in \mathbb{R}^{4nx6} \). The system becomes overdetermined for \( n \geq 2 \).

A. **Linear Least Squares**

The linear LS approach assumes an error \( \varepsilon \in \mathbb{R}^{4n} \) between predicted and actual measurements:
\[ I = VY + \varepsilon \]

Minimising the sum of the squares of errors [10] gives the parameter estimate
\[ \hat{Y} = (V^TV)^{-1}V^TI \]

B. **Non-linear Least Squares**

For the non-linear state and parameter estimator additional model equations are introduced. The model is summarized by the matrix equation
\[ M = F(S, P) \]

where \( M \in \mathbb{R}^{4n} \) is the measurement vector containing all current and voltage measurements, \( F \in \mathbb{R}^{4n} \) is the function vector that consists of the right-hand sides of equations (7)-(10) and the real and imaginary parts of the voltage variables \( V_v, V_r \) as detailed in [11].

\( F \) is a function of the states
\[ S = \{Re(V_v), Im(V_v), Re(V_r), Im(V_r)\} \]

and parameters
\[ P = \{G, B, G_1, B_1, G_2, B_2\}. \]

Assuming there is an error given by \( \mu \in \mathbb{R}^{4n} \) in the phasor measurements the estimation model is given by
\[ M = F(S, P) + \mu \]

The state and parameter estimates are computed by the iterative Newton-Raphson method [11], minimizing the sum of squares of the error \( \mu \).

C. **Total least squares**

This approach assumes the following estimation model [9]
\[ (V + E)Y = I + \epsilon \]

where \( V, Y \) and \( I \) are the matrices defined above. \( E \) models errors in the voltage, \( \epsilon \) in the current measurements. This can be re-written as
\[ [(V|I) + (E|\epsilon)] \hat{Y} = 0 \]

Parameter estimates are calculated using the singular value decomposition of the augmented matrix \( (V|I) \) as detailed in [9].

The line impedance is calculated from the estimated value of \( Y_0 \) using \( Z = 1/Y_0 \) and the impedance parameters are given by \( R = Re(Z), X = Im(Z) \); where \( R \) - resistance, \( X \) - reactance.

**CASE STUDY**

The methods outlined in the previous section have been applied to phasor data from a real power network. In this section the experimental setup and measurement results will be discussed.

**Experimental Setup**

![Figure 2](image.png)

**Figure 2.** Single line diagram of OHL in the Swedish network. A and V show the current and voltage measurement points, respectively.

The phasor data was collected at the Sweden-Poland HVDC link that passes through the Baltic Sea. A 400 kV OHL of length 100 m connects the AC/DC converter station to the Swedish grid as shown in Figure 2. Shunt capacitor banks and tuned filters installed on the line compensate reactive power and absorb harmonics, thereby reducing the impact of the converter on power quality in the main grid. To block high-frequency noise the three-phase line contains a low-pass T-filter consisting of two series inductors and a capacitor to earth. The objective was to confirm the lumped reactance value (\( X \)) at the fundamental frequency due to these two inductors using the synchronized measurements \( (V, I) \).
Three-phase voltage and current signals were sampled using two GPS-synchronized digitizer units placed at either end of the line. Their sampling rate is 20.48 kHz, locked to the nominal power system frequency of 50 Hz. Details of the instruments, voltage and current transformers are referred to [13]. The impedance measurement relies on the difference in voltage across the line, which is a small percentage of the overall voltage levels. This difference can thus be easily distorted by noise in the measured voltage phasors. Therefore observations from this experiment are also applicable to distribution networks at lower voltage as their OHL tend to be short, with low values of impedance and a small voltage across them.

**Results**

![Figure 3](image-url1)  
**Figure 3.** Histogram of 54 impedance values estimated by the linear LS method.

![Figure 4](image-url2)  
**Figure 4.** Histogram of 54 impedance values estimated by the non-linear LS method.

Overall 54 values of inductive reactance $X$ in the OHL were computed. The histograms in Figures 3 to 5 show the distributions of the estimates obtained from each method. A summary of the average values and standard deviations is given in Table 1. The manufacturers measured value of the reactance is 1.43 $\Omega$ at 50 Hz; the distribution of the linear and total LS estimates peak above this value at approximately 1.5 $\Omega$.

<table>
<thead>
<tr>
<th>Method</th>
<th>Positive sequence reactance $X$ in $\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear LS</td>
<td>1.28</td>
</tr>
<tr>
<td>Non-linear LS</td>
<td>1.48</td>
</tr>
<tr>
<td>Total LS</td>
<td>1.51</td>
</tr>
</tbody>
</table>

**Table 1.** Summary of estimated impedance values.

However, the non-linear LS estimates occupy a lower range, peaking at 1.3 $\Omega$. The standard deviation of the set of 54 values is very similar for the linear and total LS methods, with 0.11 $\Omega$ and 0.10 $\Omega$, respectively. In contrast, the non-linear state and parameter estimator resulted in a larger standard deviation of 0.14 $\Omega$.

![Figure 5](image-url3)  
**Figure 5.** Histogram of 54 impedance values estimated by the total LS method.

**Discussion**

The results show that the linear and total LS approaches yield average estimates within 10% of the known reactance value, which is acceptable as defined by the criteria in [5], and the non-linear LS estimator almost meets this threshold. The uncertainty of a given estimate is the more important metric for evaluating the methods with regards to real-time monitoring applications that have high resolution requirements (recall 0.4 % for DLR). Taking the standard deviation of the set of estimates as an indicator for the uncertainty, it becomes clear that none of the methods meets high accuracy requirements in this case study. On a relative scale the total LS method performs best, closely followed by the linear LS approach. The non-linear LS estimator gives a standard deviation which is 40% higher and hence less reliable estimates. Both the non-linear and total least squares methods have been reported to achieve better results, with estimation errors of less than 3% [14] and 7% [9], respectively. There are a number of factors which may explain why these higher accuracies and lower uncertainties could not be reproduced.
First of all it was assumed that the lumped reactance is of constant value at all times; however, it may have a current or temperature coefficient, causing some variation in the estimated values.

Secondly, the number of harmonic filtering and reactive compensation banks is not constant, but is adjusted through switching operations. This implies that the shunt parameters are changing and one interpretation of the results is that the pi circuit does not model all configurations equally well, thus causing variation in the series parameter values.

Another factor is that the impedance values depend on the small difference in sending and receiving end voltage; the accuracy of this difference relies on the level of synchronization of the measurement units. The non-uniform distribution of the impedance values is consistent with random synchronization errors. This noise is proportional to the measurements itself, which is not modelled by any of the three estimation methods and therefore not filtered optimally.

**SUMMARY**

Dynamic line rating of OHLs based on monitoring of conductor electrical resistance is a possible solution to reduce capacity constraints that prevent connection of renewable energies. In this paper three existing methods for OHL impedance parameter identification from synchronized phasor measurements have been compared by applying each to a specific case study and analyzing their relative performance. The chosen linear, non-linear and total LS estimation methods produced similar distributions of estimated values; although none met high accuracy requirements in the case study. Potential reasons for this performance are system dynamics that were not included in the model and imperfect synchronization.

In order to develop methods with a higher certainty in the identified parameter values, planned future work includes confirming the physical source as well as the distribution of errors in the phasor measurements. It is hoped that thereby existing methods can be made more robust to noise that occurs in untypical OHL parameter estimation problems.

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**REFERENCES**