PRICE-BASED CONTROL OF FLEXIBLE LOADS FOR DISTRIBUTION NETWORK MANAGEMENT

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ABSTRACT
Suitable control of flexible loads can significantly contribute to the secure and cost-efficient operation of distribution networks. Price-based control constitutes a promising alternative over traditional centralised approaches that face scalability and privacy limitations. However, naive application of price-based control leads to serious loss of diversity and demand concentration effects with adverse impacts on network constraints and losses. In order to mitigate these effects, three different measures are explored and compared in this paper. The design and performance of these measures depends on the operating properties of flexible loads; two different types are considered in this paper, namely loads with continuously adjustable power levels and loads with deferrable operation cycles. A case study on a test distribution feeder, with flexible electric vehicles and wet appliances used as representative examples of the above two flexible load types, supports the findings of this work, which was carried out in the frame of the LCNF project “Low Carbon London”.

INTRODUCTION
Controllable loads exhibit significant flexibility that can be deployed by distribution network operators (DNO) to securely and cost-efficiently operate their networks. Under the traditional control paradigm, these flexible loads will be centrally controlled by the local DNO. The loads’ will communicate their economical and technical parameters to the DNO, who will make decisions on their control based on the solution of a multi-period AC Optimal Power Flow (OPF) problem [1]. Under the envisaged large penetration of flexible loads in distribution networks however, the communication and computational scalability of centralised control approaches will be at least questionable. Furthermore, loads’ users are likely to raise privacy concerns, as they are not generally willing to disclose sensitive information and be directly controlled by an external entity.

In this context, this paper investigates an alternative, distributed control approach, optimally controlling flexible loads without requiring any centralised knowledge of their specific properties by the DNO. A suitably designed set of time- and location-specific power prices are transmitted to the loads and the latter independently determine their own control actions by minimising their payments given their own preferences and constraints.

However, naive application of such a price-based control approach, in combination with the envisaged automation in loads’ control (though smart energy management systems - EMS), leads to serious loss of diversity and demand concentration effects, as flexible loads will attempt to consume as much as possible at the lowest-priced periods of the control horizon [2]-[3]. The new demand peaks created by this concentration effect might breach the voltage and/or thermal limits of the distribution network –requiring expensive demand shedding- and increase the losses in the network as the latter are proportional to the square of the power demand. In order to avoid such concentration effects and achieve more efficient network operation, different smart measures are explored and compared in this paper. The first one, previously proposed by the authors in [2]-[3], imposes a relative flexibility restriction to loads’ EMS. In case of flexible loads with continuously adjustable power levels, this restriction corresponds to a maximum instantaneous power limit, preventing them from requesting a large proportion of their total energy requirements at the lowest-priced periods. In case of flexible loads that cannot continuously adjust their power levels but can only defer their fixed operation cycles from the time they are activated by their users, this restriction corresponds to a maximum cycle delay limit, preventing them from synchronising their operation at the lowest-priced periods.

However, imposing a flexibility restriction may be considered by the users as an external intervention in the control of their loads. This paper explores an alternative measure, where this hard flexibility restriction is replaced by a soft price signal, penalising the extent of flexibility utilised by the flexible loads. Specifically, this price penalises the square of the power demand and the duration of cycle delay of continuously adjustable and deferrable cycle loads respectively. Regarding the former type, this flexibility pricing approach is demonstrated to outperform the flexibility restriction approach in flattening the demand profile and thus achieving more cost-efficient solutions. Regarding the latter type, a third proposed smart measure randomising the flexibility price signal posted to different loads is demonstrated to bring significant additional benefits.

CASE STUDY
The performance of the price-based control approach and the smart measures to avoid demand response concentration is demonstrated through a case study on a test distribution feeder in Brixton, London, UK (Fig. 1), with relevant data obtained from [4] and considering a day-ahead control horizon with hourly resolution.

Smart charging electric vehicles (EV), which need to obtain the energy required for the desired journeys over the interval they are connected to the grid, are employed
as a representative example of continuously adjustable flexible loads. Five different scenarios are considered for the number of households owning an EV: 10%, 20%, 30%, 50% and 100%. Each EV is assumed to carry out a journey from users’ home-to-work-place and a journey in the opposite direction every day; based on the assumption of a home-charging scenario, this means that EV are connected to the grid between the end of their second and the start of their first journey of the day. Data regarding the times and electrical energy requirements of the two journeys, the maximum charging rate, the charging efficiency, as well as the EV battery energy capacity, maximum state of charge and maximum depth of discharge is obtained from [4]. Finally, the power factor of the EV chargers is assumed equal to 0.9 lagging.

![Image of Test distribution feeder](image)

Figure 1: Test distribution feeder

Wet appliances (WA), including dishwashers, washing machines and integrated washer-dryers, with delay functionality are employed as a representative example of deferrable cycle flexible loads. The assumed numbers of different types of WA at each bus are obtained from [4]. Five different scenarios are considered for the maximum cycle delay limit set by their users: 4h, 8h, 12h, 16h and 20h. Data regarding the cycle demand profiles and activation time distributions of the three different WA types is obtained from [5]. Finally, the power factor of WA is assumed equal to 0.9 lagging.

Without application of load control in WA operation, WA initiate their operation cycles once they are activated by their users. Without application of control in EV charging, EV start charging immediately after they are connected to the grid (after their second journey) with their maximum charging rate until they are fully charged. Given that most users return home during evening hours 17-20 when the non-EV demand peak occurs, the overall peak demand of the feeder is significantly increased. As a result, the power flow at the top feeder section breaches its thermal capacity limit in every EV penetration scenario. In order to securely operate the network, the DNO needs to shed some demand (Table 1); as expected, the demand shed increases with the EV penetration.

<table>
<thead>
<tr>
<th>No EV</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>50%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.17</td>
<td>0.96</td>
<td>1.89</td>
<td>3.73</td>
<td>9.16</td>
</tr>
</tbody>
</table>

### PRICE-BASED CONTROL

In order to relieve the above network constraint without the need for expensive demand shedding, the DNO may decide to deploy the flexibility in EV and WA operation. As discussed in the Introduction, a price-based control approach is employed for the coordination of these flexible loads.

A set of location- and time-specific prices for active and reactive power $\lambda_{b,t}$ and $\mu_{b,t}$, $\forall b \in B, \forall t \in T$ (where $b$ and $B$ represent the index and set of network buses and $t$ and $T$ represent the index and set of time periods) are transmitted to the loads’ EMS. In line with existing dynamic pricing schemes, these prices are designed in such a way, so as to incentivise the loads to shift their operation towards off-peak periods and thus relieve network constraints and reduce network losses. A simple approach that the DNO can follow to determine such prices is by solving a multi-period ACOPF problem considering a projection of the inflexible demand profile; the values of the resulting Lagrangian multipliers associated with the active and reactive power balance at each bus and time period physically represent the respective active and reactive power prices.

Given these prices, the EMS of each load $i$ connected to bus $b$ independently determines its own control actions by solving their payment minimisation problem. Regarding continuously adjustable loads, this problem is formulated as (where $p_{i,t}$ is the active power of load $i$ at hour $t$, $\overline{P_i}$ is the maximum active power limit of load $i$, $E_i$ is the total energy requirement of load $i$ and $pf_i$ is the power factor of load $i)$:

$$\min_{p_{i,t}} \sum_{t \in T} (\lambda_{b,t} + \mu_{b,t} \star \tan^{-1} pf_i) \star p_{i,t}$$  \quad (1)

Subject to:

$$0 \leq p_{i,t} \leq \overline{P_i}, \forall t \in T$$  \quad (2)

$$\sum_{t \in T} P_{i,t} = E_i$$  \quad (3)

Regarding deferrable cycle loads, their payment minimisation problem is formulated as (where $P_{i,t}$ is the active power at step $t$ of the cycle of load $i$, $T_{i}^{dur}$ is the duration of the cycle of load $i$, $T_{i}^{act}$ is the time load $i$ is activated by its users, $t_{i}^{in}$ is the time the cycle of load $i$ is actually initiated and $d_i$ is the maximum cycle delay limit set by the users of load $i)$:

$$\min_{t_{i}^{in}} \sum_{t=1}^{T_{i}^{dur}} (\lambda_{b,t} + \mu_{b,t} \star \tan^{-1} pf_i) \star P_{c,i}$$  \quad (4)
Subject to:

\[ t_i^{\text{act}} \leq t_i^{\text{in}} \leq t_i^{\text{act}} + d_i \quad (5) \]

For both types of flexible loads, application of this price-based control approach leads to serious loss of diversity and demand concentration effects, as flexible loads will attempt to consume as much as possible at the same late night/early morning hours exhibiting the lowest prices due to their low inflexible demand levels. As explained in [2]-[3], this effect is mathematically justified by the fact that the optimal demand response of both flexible load types is a discontinuous function of the prices.

Fig. 2 demonstrates this effect for a case where control is applied to flexible EV only. The size of the concentration effect is increased with higher EV penetrations, and under the 50% and 100% penetration scenarios, the power flow at the top feeder section breaches its thermal capacity limit, leading to demand shedding (equal to 1.61MWh and 8.99MWh respectively).

![Figure 2: Power flow on top feeder section under price-based control of EV](image)

Fig. 3 demonstrates this effect for a case where control is applied to flexible WA only and the penetration of EV is equal to 10%. The size of the concentration effect is increased with larger maximum cycle delays, and under the d=20h scenario, the power flow at the top feeder section breaches its thermal capacity limit, leading to demand shedding of 0.72MWh.

![Figure 2: Power flow on top feeder section under price-based control of WA (10% EV penetration)](image)

**SMART MEASURES AGAINST DEMAND CONCENTRATION**

**Relative flexibility restriction**

The size of the demand concentration effect is enhanced when the flexibility of the loads is larger. Regarding continuously adjustable loads, their flexibility is associated with their maximum power limit; a larger maximum power limit enables them to acquire larger proportion of their energy requirements at the lowest-priced periods and thus aggravates the concentration effect. Regarding deferrable cycle loads, their flexibility is associated with their maximum cycle delay; a larger maximum cycle delay limit enables them to execute their cycles over a wider time window. Given that different loads are activated at different periods, a larger maximum cycle delay limit increases the number of loads executing their cycles at the lowest-priced time periods and thus aggravates the concentration effect.

Driven by this observation, the first measure, proposed by the authors in [2]-[3], transmits a relative flexibility restriction signal \( \omega \) to the loads’ EMS, which represents the fraction of available flexibility that can be utilised. For continuously adjustable loads, \( \omega \) is interpreted by their EMS as their allowed maximum power limit as a fraction of the respective nominal one. Application of this measure means that constraint (2) of their payment minimisation problem is transformed to:

\[ 0 \leq p_{i,t} \leq \omega \cdot p_{i,t}^{\text{in}}, \forall t \in T \quad (6) \]

For deferrable cycle loads, \( \omega \) is interpreted by their EMS as their allowed maximum cycle delay limit as a fraction of the respective set by their users. Application of this measure means that constraint (5) of their payment minimisation problem is transformed to:

\[ t_i^{\text{act}} \leq t_i^{\text{in}} \leq t_i^{\text{act}} + \omega \cdot d_i \quad (7) \]

**Flexibility pricing**

Imposing such flexibility restrictions may not be deemed acceptable by the users, as they may consider it as a direct intervention of the DNO in the control of their assets. An alternative measure explored in this paper addresses this concern by replacing this hard restriction by a soft price signal \( \alpha \), penalising the extent of flexibility utilised.

For continuously adjustable loads, \( \alpha \) is interpreted by their EMS as a penalty on the square of their power demand, and thus indirectly limits the maximum power requested. Application of this measure means that the objective function (1) of their payment minimisation problem is transformed to:

\[ \min_{p_{i,t}} \sum_{t \in T} \left( \lambda_{b,t}^2 + \mu_{i,t}^2 \cdot \tan^{-1} p_{i,t} \right) \cdot p_{i,t} + \alpha \cdot p_{i,t}^2 \quad (8) \]

For these loads, apart from the above acceptability advantage, this flexibility pricing measure exhibits an optimality advantage over the flexibility restriction...
approach. Specifically, the objective function of the payment minimisation problem becomes quadratic and thus the optimal response can admit a larger number of values in the interior of the loads’ feasible operation domain, resulting in a better demand flattening effect.

For deferrable cycle loads, \( \alpha \) is interpreted by their EMS as a penalty on their cycle delay. Application of this measure means that the objective function (4) of their payment minimisation problem is transformed to:

\[
\min_{t_i} \sum_{t=1}^{d_{\text{dur}}} \left( \lambda_{b_i} + \mu_{i} + \alpha * \tan^{-1} p_{t_i} * \frac{P_{t_i}}{\tan^{-1} p_{t_i}} \right) *
\]

\[
\alpha \geq \alpha^* + \sigma * x_i \tag{10}
\]

where \( \alpha^* \) and \( \sigma \) denote respectively the mean value and standard deviation of the distribution of flexibility prices.

**Randomised flexibility pricing**

Deferrable cycle loads are characterised by discrete operation nature since their control lies in deciding the (integer) number of time periods their cycle will be deferred by. This means that application of a uniform flexibility restriction or flexibility price cannot sufficiently limit the concentration effect. This can be better understood by considering the extreme example where all deferrable cycle loads in the network have identical operating parameters; irrespectively of the value of \( \omega \) or \( \alpha \), all loads will initiate their cycle at the same period and the concentration effect cannot be avoided.

In this context, the third proposed measure introduces diversification in the response of deferrable cycle loads by diversifying the flexibility price \( \alpha \). Since the DNO does not have any information on the parameters of the loads to drive the specifics of such diversification, a randomisation approach is proposed. A vector \( X \) is derived by a random number generator, the size of which is equal to the number of deferrable cycle loads in the network and its elements \( x_i \) take random values following the standard normal distribution (standard distribution with zero mean and unity standard deviation). The randomised flexibility price posted to load \( i \) is given by:

\[
\alpha_i = \alpha^* + \sigma * x_i
\]

Measures tuning

In general, relatively large values of \( \omega \) and relatively small values of \( \alpha \) and \( \sigma \) may not sufficiently limit the loads’ flexibility to concentrate their demand at the lowest-priced periods, while relatively small values of \( \omega \) and relatively large values of \( \alpha \) and \( \sigma \) may limit excessively their flexibility and thus prevent them from shaving the peaks and filling the off-peak valleys of the inflexible demand profile. A suitable value of \( \omega, \alpha \) and \( \sigma \) should be employed by the DNO to achieve an effective trade-off between these two effects, leading to a flatter demand profile and more efficient operation. Such suitable values will depend on the correlation between the characteristics of the flexible loads population (number, nominal flexibility and diversity) and the temporal variation of inflexible demand. For a certain inflexible demand profile, a larger number and flexibility and a poorer diversity of the flexible loads population will result in a smaller value of the most suitable \( \omega \) and a larger value of the most suitable \( \alpha \) and \( \sigma \). Tables 2 and 3 present examples of this suitable tuning in the case where control is applied to EV and WA respectively, justifying the above.

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>50%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha ) (p/kWh^2)</td>
<td>0.0005</td>
<td>0.001</td>
<td>0.0015</td>
<td>0.002</td>
<td>0.005</td>
</tr>
</tbody>
</table>

| \( \sigma \) (p/h) | 0.0001 | 0.0002 | 0.0002 | 0.0002 | 0.0003 |
| \( \sigma \) (p/h) | 0.00005 | 0.0002 | 0.0003 | 0.00005 | 0.0008 |

In practical terms, it is envisaged that the DNO will perform the tuning of these parameters by trying out a number of different values and employing suitable learning algorithms. Finally, regarding the randomisation approach, the most suitable value of the uniform flexibility price is employed as the mean value \( \alpha^* \) of the distribution of flexibility prices.

**COMPARISON**

This section compares the results obtained under different control approaches in the examined study. The values of Tables 2 and 3 are employed when the measures presented in the previous section are applied.

**EV control**

As EV are continuously adjustable loads, application of a uniform flexibility restriction or price limits high demand levels from each individual EV. As justified by our simulations, this means that the randomisation measure discussed above does not generally bring additional benefits and thus is not examined further in this subsection.

As demonstrated in Fig. 4, both flexibility restriction and flexibility pricing measures mitigate the response concentration effect and achieve a flatter total demand profile than the basic price-based control approach (price-based control without additional measures). As a result, the thermal constraint of the top feeder section is not violated and demand shedding is not required in any of the examined EV penetration scenarios. Furthermore, the flatter demand profile is translated into a reduction of network losses, given that the latter are proportional to the square of the power demand, as observed in Fig. 5 (100% EV penetration scenario constitutes an exception.

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**Table 2:** Selected values of \( \omega \) and \( \alpha \) for EV control under different EV penetration scenarios

- 10%: 0.4
- 20%: 0.35
- 30%: 0.3
- 50%: 0.25
- 100%: 0.2

**Table 3:** Selected values of \( \alpha \) and \( \sigma \) for WA control under different WA maximum delay scenarios

- \( d=4h \): 0.0001
- \( d=8h \): 0.0002
- \( d=12h \): 0.0002
- \( d=16h \): 0.0002
- \( d=20h \): 0.0003

- \( \sigma \) (p/h): 0.00005
- \( \sigma \) (p/h): 0.0003
- \( \sigma \) (p/h): 0.00005
- \( \sigma \) (p/h): 0.0008

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CIRED 2015

23rd International Conference on Electricity Distribution

Lyon, 15-18 June 2015

Paper 0808

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as under basic price-based control the significant required demand shedding reduces losses greatly.
For the reason explained in the previous section, the flexibility pricing measure performs better that the flexibility restriction measure in flattening the demand profile and reducing network losses.

WA control
Our simulations indicate that the performance of uniform flexibility restriction and uniform flexibility pricing measures is similar. Therefore, only uniform and randomised flexibility pricing measures are explored in this sub-section.
As demonstrated in Fig. 6, both measures limit the response concentration effect and achieve a flatter demand profile than the basic price-based control approach. As a result, the thermal constraint of the top feeder section is not violated and demand shedding is not required in any of the examined WA maximum delay scenarios. Furthermore, the flatter demand profile is translated into a reduction of network losses, as observed in Fig. 7 (d=20h scenario constitutes an exception as under basic price-based control the required demand shedding reduces losses).
For the reason explained in the previous section, the randomised flexibility pricing measure performs better that the uniform flexibility pricing measure in flattening the demand profile and reducing network losses.

ACKNOWLEDGEMENTS
The work presented in this paper was carried out in the frame of the LCNF project “Low Carbon London”.

REFERENCES