ADAPTIVE WEIGHTED LEAST SQUARES-BASED ALGORITHM TO ESTIMATE SYNCHRONIZED MEASUREMENTS OVER WIDE FREQUENCY RANGE

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ABSTRACT

This paper proposes a frequency-adaptive weighted least squares-based algorithm for accurate identification and estimation of synchronized measurements over wide frequency range. A new mathematical formulation is performed to estimate all synchronized components of a single-phase voltage or current input signal when spurious frequency such as harmonics and off-nominal frequency, within the range of ±5 Hz around the fundamental frequency, are presented. The proposed algorithm decouples local system frequency estimation and synchronized components tracking, in which the static off-nominal frequency is estimated from the classical method based on the rate of change of phase angle relative to the offset from fundamental frequency whose outcome is used to calculate a new sampling angle that is utilized in the computation of the synchronized components. Several noisy signals are applied to evaluate the algorithm's performance where the results obtained demonstrate the capability of the proposed algorithm in dealing efficiently with very corrupted signals. The algorithm is envisaged for M-Class Phasor Measurement Unit (PMU) aiming to improve the power quality issue of power distribution systems.

INTRODUCTION

The long-awaited energy efficiency is one of the main premises envisaged for the smart distribution systems. In this context, new possibilities for power delivery and consumption, new paradigms of communication infrastructures and advanced metering might be created to allow the evolution of electric distribution grids from passive to active networks. The Advanced Phasor Measurement Units (PMUs) and other GPS-enabled Intelligent Electronic Devices (IEDs-GPS) should play a critical role in the smart distribution systems being envisioned, because the synchronized measurements provided by these units may ensure real-time and synchronized situational awareness, resilient operation against disturbances in presence of bidirectional power-flow and reliable integration of micro-grids with considerable rate of decentralized production. The reliability of the synchronized measurement systems [1] is another major factor to boost its deployment in smart power distribution environment.

Unfortunately, most of the existing commercial PMUs and IEDs-GPS have severe problems to comply with the IEEE Std. C37.118.1 [2] during dynamic conditions especially under wide range frequency variation, therefore high measurement errors, especially large phase lag, can be obtained in the estimated voltage and current phasors. It is important to comment that the topology of power distribution systems with shorter line lengths, the potential presence of high harmonic content, and frequency variation caused by defaults will contribute with additional challenges for the synchronized devices placed at these systems. In this context, new algorithms must be proposed to avoid that the above reported facts impose constraints in the applications of PMUs and IEDs-GPS on distribution level.

The most popular algorithms used to estimate synchronized measurements are based on the Discrete Fourier Transform (DFT) matched with the fundamental frequency [3], however this powerful technique suffers with frequency drift that provokes the pernicious leakage phenomena. Aiming to overcome the underlying problem, weighing windows are applied in the discrete-time domain to produce fewer ripples in frequency domain caused by the abrupt truncation of the signal outside of the required observation interval. Generally, cosine windows such as Hanning, Hamming, Blackman, and Flat-top have been used to alleviate the impact of asynchronous condition. Other methods have been also applied, e.g., Digital Phase-Locked Loop (DPLL) to obtain sampling clock synchronization related to the actual frequency [4], but this method may face difficulties with clock resolution and noise rejection.

Non-DFT-type phasor estimators have also been proposed in the literature [5]-[7], however the weighted least squares approach (WLS) emerges as an efficient method to provide accurate estimation of the parameters of interest. In [8], a weighted least error squares algorithm with variable data window for signal phasor estimation with regard to distance relaying application is proposed. A WLS synchrophasor estimator based on the Maximum Sidelobe Decay (MSD) or the Minimum Sidelobe Level (MSL) windows is presented in [9]. In [10], a Taylor-WLS algorithm based on the generalization to both windowing and dynamic waveform model of the three-parameter sine-fitting has been implemented. Recursive least squares (RLS) and least mean squares (LMS) algorithms are proposed in [11], which goal is to provide dynamic estimates of voltage phasors. Recursive Least Squares (RLS) algorithm with modified covariance matrix has been proposed in [12] to improve the convergence feature.

In this work, a practical and accurate frequency-adaptive algorithm based on the WLS approach is proposed for reliable identification and estimation of synchronized measurements over wide frequency range. A new mathematical formulation is performed to estimate all synchronized components of a single-phase voltage or current input signal when spurious frequency such as harmonics and off-nominal frequency, within the range of ±5 Hz around the fundamental frequency, are presented. The maximum number of state variables is limited by the desired harmonic component imposed by the 3-dB cut-off frequency, related to the analog anti-aliasing lowpass filter, added with the DC offset component. The goal is to have full signal representation and a computational burden reduction. The sampling clock is locked to the fundamental frequency and the actual frequency is estimated from the rate of change of phase angle relative to the offset from nominal frequency whose outcome is used to compute a new sampling angle to be utilized in the computation of the synchronized components. An extensive range of synthetized noisy signals are applied to evaluate the algorithm's performance whose results demonstrate the capability of the proposed algorithm in dealing efficiently with corrupted signals. The algorithm is envisaged for M-Class PMU model to improve the power quality issue of power distribution systems.

PROPOSED ALGORITHM

In this section, a nominal frequency of f_o = 60-Hz is considered, however the procedures could also be applied to 50-Hz signals. A fixed sampling rate corresponding to N_s = 24 samples/cycle of the nominal frequency is used resulting a Nyquist frequency of 12 samples/cycle (*i.e*, 720 Hz for a 60-Hz system). The input signal is bandlimited to 500 Hz (3-dB cut-off frequency) which is below the required value to obey the Nyquist criterion.

Frequency estimation

For proper functionality, the proposed algorithm requires accurate knowledge of the local system frequency. A lot of methods with regard to frequency estimation can be found in the literature [13]-[14]. Based on the state-of-art proposed by [3], a decoupled frequency estimation process has been implemented using also WLS approach due to its good performance even when the phase angle computation are corrupted by Gaussian noise (ε). The frequency estimator uses sampling clock locked to the nominal frequency (ΔT_o) and the frequency deviation (Δf_o) is estimated from the rate of change of phase angle relative to the offset from nominal, i.e., the phase angle is the integral of the frequency. Taking advantage of this feature, two additional state variables related to the initial value of the angle (ϕ_0) and the rate of change of frequency (ROCOF) (f') can be estimated together with the frequency deviation. The complete representation of the overdetermined linear system is given by

$$\begin{bmatrix}\phi_0\\ \phi_1\\ \phi_2\\ \vdots\\ \phi_r\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 1 & 2\pi\Delta T_o & \pi\Delta T_o^{\,2}\\ 1 & 4\pi\Delta T_o & \pi 2^2\Delta T_o^{\,2}\\ \vdots & \vdots & \vdots\\ 1 & 2\pi r\Delta T_o & \pi r^2\Delta T_o^{\,2}\end{bmatrix}\times\begin{bmatrix}\phi_0\\ \Delta f_o\\ f'\end{bmatrix}+\begin{bmatrix}\varepsilon_{\phi 0}\\ \varepsilon_{\phi 1}\\ \varepsilon_{\phi 2}\\ \vdots\\ \varepsilon_r\end{bmatrix} \quad (1)$$

being the r-index the amount of computed phase angles. In matrix notation, (1) can be rewrite as

$$[\phi] = [A][F] + [\varepsilon] \quad (2)$$

hence, the state vector $[F]$ is calculated by WLS approach as (3), being W the covariance matrix related to errors

$$[\hat{F}] = [A^T W^{-1} A]^{-1} A^T W^{-1} [\phi]. \quad (3)$$

Synchronized components tracking

The mathematical formulation of the synchronized components tracking is shown as follows: let the single-phase sinusoidal signal *(in pu*) be given by

$$x(t) = x_{DC} + X_1\cos(2\pi f_m t + \varphi_1) + \sum_{m=2}^{k} X_m cos(2\pi m f_m t + \varphi_m) \quad (4)$$

X_1 and X_m being the peak amplitude of fundamental and harmonic components, respectively; φ_1 and φ_m being the phase angle of the fundamental and the harmonics during the observation interval, respectively; x_{DC} is the DC offset; k is the highest harmonic present in the signal; and f_m is the off-nominal frequency provided by the frequency estimation process.

Taking (4) at each fixed sampling time $n\Delta T_o$ matched to the nominal system frequency, provides

$$x[n] = x_{DC} + X_1\cos(2\pi n f_m \Delta T_o + \varphi_1) + \sum_{m=2}^{k} X_m\cos(2\pi n m f_m \Delta T_o + \varphi_m) \quad (5)$$

that is,

$$x[n] = x_{DC} + X_1\cos(n\gamma + \varphi_1) + \sum_{m=2}^{k} X_m cos(nm\gamma + \varphi_m) \quad (6)$$

being $\gamma=2\pi f_m/N_s f_o$ locked to the actual f_m frequency.

Applying the full-cycle DFT for the sequence $x[n]$ in (6), and separating the real and imaginary parts of the signal a special analysis can be performed. Taking into account the imaginary part of the DFT one can observe

$$\frac{\sqrt{2}}{N_s}\sum_{n=0}^{N_s-1} x[n]\sin(n\theta) = \frac{\sqrt{2}}{N_s}\sum_{n=0}^{N_s-1} x_{DC}\sin(n\theta) \quad (7)$$

$$+\frac{\sqrt{2}}{N_s}\sum_{n=0}^{N_s-1}[X_1\cos(n\gamma+\varphi_1)\sin(n\theta)] \quad (8)$$

$$+\frac{\sqrt{2}}{N_s}\sum_{n=0}^{N_s-1}[X_m\cos(nm\gamma+\varphi_m)\sin(n\theta)] \quad (9)$$

$$(m = 2, \ldots, k)$$

where $\theta = 2\pi/N_s$ the nominal sampling angle.

Applying the following trigonometric identity

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \qquad (10)$$

One can rewrite (8) and (9) as

$$\frac{\sqrt{2}X_1}{N_s}\cos(\varphi_1)\sum_{n=0}^{Ns-1}[\cos(n\gamma)\sin(n\theta)] - \frac{\sqrt{2}X_1}{N_s}\sin(\varphi_1)\sum_{n=0}^{Ns-1}[\sin(n\gamma)\sin(n\theta)] \qquad (11)$$

$$\frac{\sqrt{2}X_m}{N_s}\cos(\varphi_m)\sum_{n=0}^{Ns-1}[\cos(nm\gamma)\sin(n\theta)] - \frac{\sqrt{2}X_m}{N_s}\sin(\varphi_m)\sum_{n=0}^{Ns-1}[\sin(nm\gamma)\sin(n\theta)] \qquad (12)$$
$$(m = 2, \ldots, k)$$

It is possible to note from (7), (11), and (12) that the phasor estimation problem can be reformulated in the least squares sense to find the state vector using the off-nominal samples through a set of h overdetermined linear equations, given by (13). A simple mathematical modification is performed to ensure a correct correlation for the DC offset component

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_h \end{bmatrix} = H \times \begin{bmatrix} x_{DC} \\ |X_1|\cos(\varphi_1) \\ |X_2|\cos(\varphi_2) \\ \vdots \\ |X_k|\cos(\varphi_k) \\ |X_1|\sin(\varphi_1) \\ |X_2|\sin(\varphi_2) \\ \vdots \\ |X_k|\sin(\varphi_k) \end{bmatrix} + \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_h \end{bmatrix} \qquad (13)$$

where H is the coefficient matrix given by

$$H = \sqrt{2}\begin{bmatrix} \frac{1}{\sqrt{2}} & \cos(0) & \ldots & \cos(k0) & -\sin(0) & \ldots & -\sin(k0) \\ \frac{1}{\sqrt{2}} & \cos(\gamma) & \ldots & \cos(k\gamma) & -\sin(\gamma) & \ldots & -\sin(k\gamma) \\ \frac{1}{\sqrt{2}} & \cos(2\gamma) & \ldots & \cos(2k\gamma) & -\sin(2\gamma) & \ldots & -\sin(2k\gamma) \\ \vdots & \vdots & \ldots & \vdots & \vdots & \ldots & \vdots \\ \frac{1}{\sqrt{2}} & \cos(h\gamma) & \ldots & \cos(hk\gamma) & -\sin(h\gamma) & \ldots & -\sin(hk\gamma) \end{bmatrix}$$

being the h-index the amount of samples. Several elements of H matrix are numerically identical, thus a formation law to reduce the calculation effort has been performed. Considering h equal to N_s-1, a reduction of 40.2% of calculation burden can be obtained. In practice, the length of the measurements vector $[x]$ is equal to N_s samples, however the formulation can also be applied for fractional data windows provided that the amount of samples be greater than the number of variables to be determined. Furthermore, the maximum number of state variables, equal to $2k$+1, is related to the 3-dB cut-off frequency, in other words, for 500 Hz the maximum off-nominal harmonic order component that can be estimated is equal to 8 providing a sate vector with length equal to 17. A matrix notation can be formulated as (14), in which $[X]$ is the state vector, and the vector $[\varepsilon]$ has been added to represent the out-of-band Gaussian noise related to the quantization error

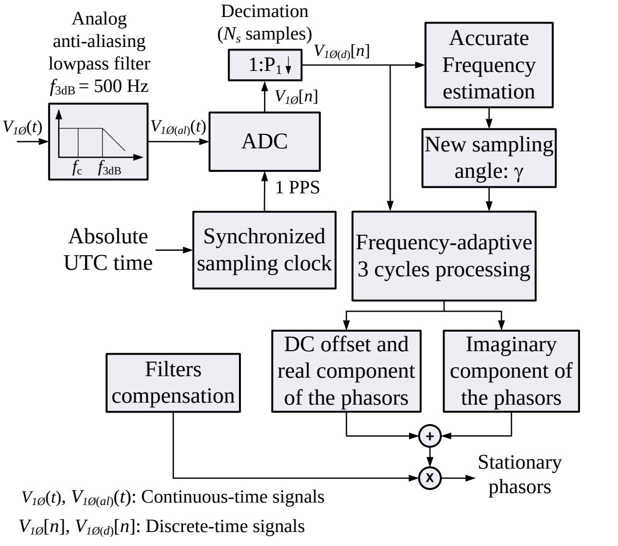

Fig.1. Practical frequency-adaptive weighted least squares based algorithm for M-Class PMU model.

$$[x] = [H][X] + [\varepsilon]. \qquad (14)$$

Thus, in the weighted least squares sense the solution for the overdetermined linear system (14) is given by

$$[\hat{X}] = [H^T W^{-1} H]^{-1} H^T W^{-1}[x] \qquad (15)$$

where W represents the diagonal covariance matrix related to the error vector. The elements of the W matrix are intrinsically associated with the accuracy of the synchronized device.

Fig. 1 depicts the complete diagram of the proposed algorithm. An analog anti-aliasing lowpass filter based on the Butterworth approximation is employed to the signal prior to any processing inside the proposed algorithm. Butterworth has been chosen due to its smoother response in both the passband and stopband regions, and it provides a more linear phase response [15]. It is noteworthy to point out that the phase shifts introduced by this filter related to the estimated frequency and their harmonics need to be compensated in the proposed algorithm. A digital decimation filter has also been included to ensure a concatenation with the anti-aliasing lowpass filter for a high ADC sampling rate condition.

PERFORMANCE EVALUATION OF THE PROPOSED ALGORITHM

This section presents the simulation and result analysis for a power system with 60-Hz nominal frequency. The proposed algorithm has been implemented in MATLAB 2014a environment. Test cases are performed in four parts: the starting point is off-nominal single-phase sinusoidal test signal (*in pu*) corrupted with small DC offset, high signal-to-noise ratio (SNR) related to small errors imposed by the ADC quantization process, and harmonic components until the 8*th* order. For this test case, the goal is to evaluate and compare the off-nominal phasor estimates with classical window methods (Rectangular, Hamming, and Blackman). In the second, it is assumed an off-nominal input signal also corrupted by high order spurious harmonic components, however with a high superimposed DC offset. In the third and four cases, the performance of the proposed algorithm is evaluated under high out-of-band Gaussian noise. For all test cases the actual frequency has been tracked by the frequency estimation process whose outcome is used to modify the parameters of the synchronized components tracking.

The test signal employed for the *case I* has a static frequency of 64 Hz and voltage amplitude of 1%, 100%, 3.5%, 6%, 1.3%, 7.5%, 1%, 7%, 1.2%, related to the DC offset until the 8*th* harmonic component, respectively. A SNR=77 dB has been used, and a Gaussian random noise with zero mean and a standard deviation of 0.01 radian has been used to represent the errors in the phase angle estimates in the frequency estimation process. Performing 1000 Monte Carlo trials the estimated frequency deviation is around 3.9993 Hz. Total Vector Error (TVE) metric is the measure of error between the theoretical synchrophasor value of the signal being measured and the synchrophasor estimate, as (16). It must be less than 1% to comply with the IEEE Std. C37.118.1 [2].

$$TVE(k) = \frac{\left|\vec{X}_{estimate}(k) - \vec{X}_{theoretical}(k)\right|}{\left|\vec{X}_{theoretical}(k)\right|} \times 100\% \quad (16)$$

Fig. 2 shows the TVE results given by the proposed algorithm for each component of the test signal which values satisfactorily fulfills the requirements of the standard. Fig. 3 shows the performance of the classical windowed DFT. Clearly, the combination of DC offset, harmonic components and high frequency deviation will cause severe estimation errors for the classical DFT method. The test signal used in the *case II* has a static frequency of 60.05 Hz and high DC offset around 10% and SNR=60 dB. The fundamental until the 8*th* harmonic component have voltage amplitude of 100%, 3%, 7%, 1%, 8%, 1%, 7.5%, 1%, respectively. For this test case, the estimated frequency deviation is around 0.04991 Hz and Fig. 4 illustrates the TVE error for each off-nominal component present in the signal.

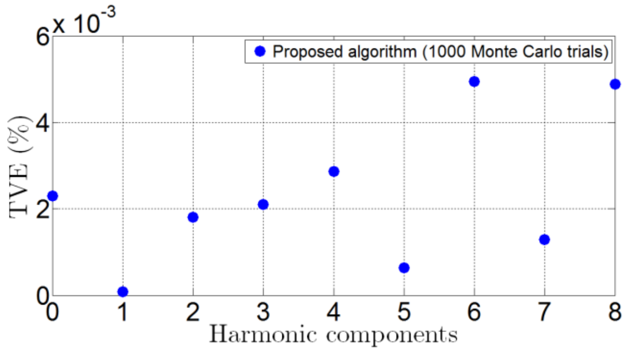

Fig.2. Total Vector Error for each harmonic component of the noisy signal at estimated frequency of 63.9993 Hz.

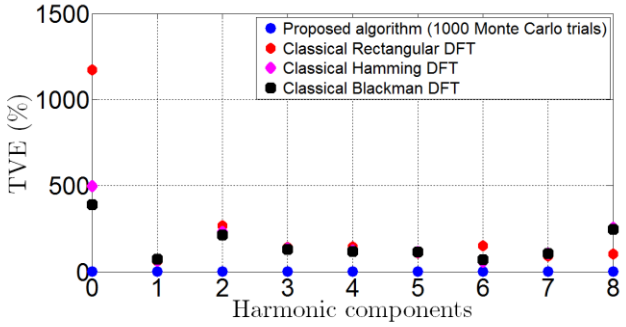

Fig.3. Performance of proposed algorithm and the classical windowed Discrete Fourier Transform.

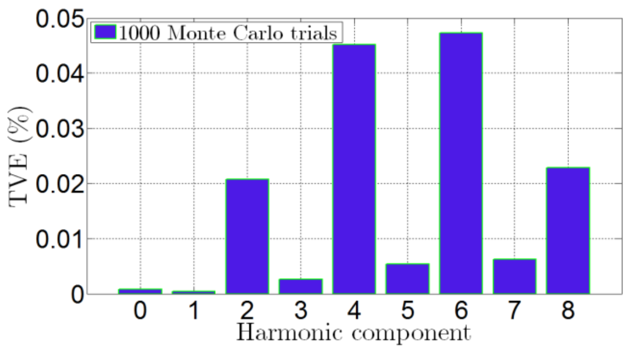

Fig.4. Total Vector Error for each harmonic component for a signal with high DC offset and SNR=60 dB at estimated frequency of 60.04991 Hz.

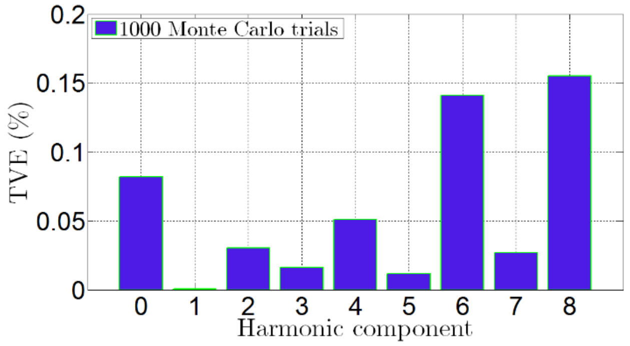

Fig.5. Total Vector Error for each harmonic component for a signal with a SNR=46 dB at estimated frequency of 59.9505 Hz.

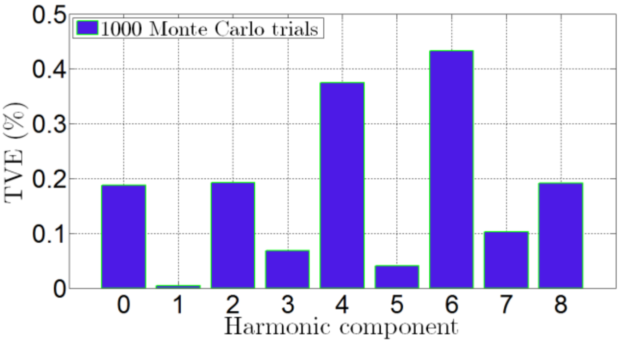

Fig.6. Total Vector Error for each harmonic component for a signal with a SNR=34 dB at estimated frequency of 59.9505 Hz.

One can observe that even increasing the DC offset and the noise the algorithm provides accurate results. The test signal used for the test cases *III* and *IV* have a static frequency of 59.95 Hz and voltage amplitude of 2%, 100%, 4%, 7%, 1.8%, 8.5%, 2%, 7.5%, and 1.5%, respectively. High out-of-band Gaussian noise causing a SNR=46 dB and SNR=34 dB for the test cases *III* and *IV* are considered, respectively. The estimated frequency deviation is closely to 0.9505 and the Figs.5-6 display the estimated off-nominal components. It is possible to note that the errors tend to increase, however the results obtained are still in accordance with the IEEE Std. C37.118.1. For a SNR below 34 dB, the estimates come dangerously close to the threshold established by this standard.

CONCLUSION

In this work, a frequency-adaptive weighted least squares-based algorithm for accurate identification and estimation of synchronized measurements under frequency variation within the range of ±5 Hz offset-nominal has been proposed. The algorithm decouples local system frequency estimation and synchronized components tracking whose outcome of the first is used to modify the computation of the second. Four test cases employing input signals corrupted by DC offset, harmonic components, frequency deviation, and out-of-band Gaussian noise have been realized to evaluate the performance of the algorithm which results satisfactorily fulfills the requirements of the IEEE Std. C37.118.1 as regards accuracy, thus the proposed algorithm is envisaged for M-Class PMU model aiming to improve the power quality issue of power distribution systems.

Acknowledgments

This work was carried out as part of the SOGRID project (www.sogrid.com), co-funded by the French agency for Environment and Energy Management (ADEME) and developed in collaboration between participating academic and industrial partners. The second author would like to thank the CAPES Foundation of Brazil for the valuable support.

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