PERFORMANCE ANALYSIS FOR NON-LINEAR LOAD MODELLING WITH FREQUENCY COUPLING ADMITTANCE MATRICES

Anke FROEBEL
Otto-von-Guericke University
Magdeburg – Germany
anke.froebel@ovgu.de

Ralf VICK
Otto-von-Guericke University
Magdeburg – Germany
ralf.vick@ovgu.de

ABSTRACT

This paper focuses on a method for modelling small non-linear loads by means of frequency coupling admittance matrices. Based on measurements the used technique delivers equivalent Norton admittance matrices of the actual non-linear device. Various types of LED lamps serve as test objects. It is shown how the absorbed currents vary in dependency of amplitude and phase of voltage harmonic distortion. The resultant changes of single matrix elements are analysed and the model performance is evaluated.

INTRODUCTION

Traditionally, in harmonic studies the impact of small non-linear loads has been either ignored or simplified by modelling group effects. As the number of such loads has been increasing for years and they now form a large part of the electrical load in power systems, their harmonic effects become more and more detrimental. The decentralized nature of non-linear devices such as computers, televisions, phone chargers, energy saving as well as LED lamps, etc. throughout distribution networks requires the consideration of interaction phenomena and thus complicates system wide mitigation measures.

In the following a method for modelling non-linear loads by means of frequency coupling admittance matrices is focused. Based on measurements the used technique delivers equivalent Norton admittance matrices of the actual non-linear device. To get the necessary parameters sequentially small harmonic voltage distortions are applied to the load terminals and the occurring currents are measured. The method treats the non-linear load as a "black box". Knowledge about operation principles and circuit design is not necessary.

After an introduction to the modelling technique, an experimental setup is presented. Different types of LED lamps serve as test objects. Including admittance tensors the resulting linearized models in harmonic domain consider phase dependency of voltage harmonics. As phase, magnitude, input and output frequency dimensions are freely selectable, testing time can be lengthy. If the phase of the applied voltage distortion varies from 0 to 2π, the corresponding element of the admittance matrix ideally changes in the form a double circle locus. Thus, theoretically, the terms of the tensor can be found using only two admittance points. But this method is prone to experimental error [1]. Therefore, results from different numbers of measuring points are compared to find a good compromise between measuring effort and model performance.

With growing level and for certain frequencies of the applied distortion the behaviour of the lamps becomes highly non-linear. As the model implies linearization around a working point, limits for its application concerning the harmonic voltage distortion are derived.

FREQUENCY COUPLING ADMITTANCE MATRIX

Basic Model

In [2] it is proposed to model a non-linear load by means of a "crossed-frequency" admittance matrix $\underline{\mathbf{Y}}$. As shown in equation (1) its elements $\underline{Y}_{kj}$ relate harmonic currents of certain order (k) and harmonic voltages of different order (j).

$$\underline{\mathbf{I}} = \underline{\mathbf{Y}} \cdot \underline{\mathbf{V}} \tag{1a}$$

$$\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_3 \\ \\ \underline{I}_\mathrm{M} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{11} & \underline{Y}_{12} & \cdots & \underline{Y}_{1\mathrm{N}} \\ \underline{Y}_{21} & \underline{Y}_{22} & \cdots & \underline{Y}_{2\mathrm{N}} \\ \underline{Y}_{31} & \underline{Y}_{32} & \cdots & \underline{Y}_{3\mathrm{N}} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{Y}_{\mathrm{M}1} & \underline{Y}_{\mathrm{M}2} & \cdots & \underline{Y}_{\mathrm{MN}} \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_3 \\ \\ \underline{V}_\mathrm{N} \end{bmatrix} \tag{1b}$$

The parameterization is based on physical measurements. Initially, only a basic voltage, which may be only fundamental sinusoidal voltage but may also contain an amount of harmonics, is applied to the non-linear load. The absorbed current is measured. Then, the terms $\underline{Y}_{k1}$ are determined according to equation (2).

$$\underline{Y}_{k1} = \frac{\underline{I}_k}{\underline{V}_1} \tag{2}$$

Afterwards, successively, the constant fundamental voltage is superimposed with one variable harmonic at time. According to equation (3) the resulting currents are used to calculate $\underline{Y}_{kj}$ ($j \neq 1$).

$$\underline{Y}_{kj} = \frac{\underline{I}_k - \underline{Y}_{k1}\underline{V}_1}{\underline{V}_j} \tag{3}$$

The approach implies that all variations of the absorbed current in comparison with the fundamental response are

due to the added harmonic voltages only. The non-linear system is linearized around a working point namely the amplitude of the fundamental voltage.
The model is supposed to calculate the harmonic currents which are absorbed by a non-linear load depending on the harmonic distortion of the terminal voltage. It is applicable for stationary, passive loads which are supplied by a periodical voltage with constant fundamental voltage.
In comparison with common non-linear load representation the proposed method has no restrictions in case of application of pure fundamental voltage [2]. In the context of an iterative frequency domain harmonic algorithm the harmonic current absorbed can be evaluated entirely in the frequency domain. The model might be used in harmonic iteration methods which model harmonic producing devices as supply voltage dependent current sources.

Harmonic Phase and Amplitude Dependency

The prediction of the crossed-frequency admittance matrix model is far away from measured waveforms when the phase of the harmonic voltages is varied. Therefore, in [3] the method is extended by adding another dimension to the matrix, the harmonic phase. The proposed result linearizes loads according to amplitude, frequency and phase of the harmonic with regard to the fundamental.
For verification of the idea different electronic loads, a computer, a fluorescent lamp with electronic ballast and an energy saving lamp are tested. The model is obtained by superposing harmonics with an amplitude equal 10% of fundamental voltage, varying the phase in steps of 30°. If the phases of the harmonics used for testing do not coincide with those used for the modelling, linear interpolation is applied. The resulting calculated currents are evaluated and compared with measured currents in time domain. The more harmonics are used for matrix development the smaller is the error. Its maximum is in the range of 5%. Some unsatisfactory results are obtained for high content of odd harmonics.
The usability of crossed-frequency admittance matrices for network calculations in case of changing network parameters is also described in [4]. The method is used to find an equivalent for a load with high, pulsating power. The resulting model allows mathematical simulation of resistive welding machines for different power ranges. It is pointed out that the sensitivity of the model to changes of voltage harmonic phases is a serious disadvantage.

Experimental Verification

In [5] the sensitivity of the crossed-frequency admittance matrix model is evaluated for some harmonic producing loads under various waveforms of the supply voltage and compared to the performance of a constant current source model and a norton model. The latter two methods fail to include the interaction between harmonics of different order, so this feature can be emphasized as an advantage of the crossed-frequency admittance matrix model.
For comparison the models are derived for a test system supplied by a pure sinusoidal voltage. To test the performance currents are then measured for different supply voltage with harmonics up to order 11 and a total harmonic distortion of 5%. Measurement results are compared with results given by the different models. For a set of hundred test voltages the errors, its means, medians and standard deviations are calculated. The crossed-frequency admittance matrix model yields the highest mean error with highest standard deviation. It stays unclear if and how the harmonic phase and amplitude dependency of the matrix was taken into account.

Tensor Representation

To overcome the difficulty with harmonic phase dependency in [1] the use of tensors is proposed. Each element $\underline{Y}_{kj}$ can be expressed as a 2 x 2 tensor matrix (4).

$$\begin{bmatrix} \Delta I_{\text{real}} \\ \Delta I_{\text{imag}} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \Delta V_{\text{real}} \\ \Delta V_{\text{imag}} \end{bmatrix} \tag{4}$$

If the phase of the applied voltage distortion varies from 0 to 2π, the corresponding element of the admittance matrix ideally changes in the form a double circle locus which rotates around a centre $a+jb$ in the complex plane. Its radius is r. It comprises a phase shift θ of about its own axis. In this manner it is referred to the admittance at a distortion of zero degrees. For an even number N voltage phase angles, evenly spaced between 0 and 2π a, b, r and θ can be calculated according to (5)-(8). The tensor values can be derived using (9)-(12).

$$a = \frac{1}{\mathrm{N}} \sum_{n=1}^{\mathrm{N}} \mathrm{real}\left(y(n)\right) \tag{5}$$

$$b = \frac{1}{\mathrm{N}} \sum_{n=1}^{\mathrm{N}} \mathrm{imag}\left(y(n)\right) \tag{6}$$

$$r = \frac{1}{\mathrm{N}} \sum_{n=1}^{\mathrm{N}} \mathrm{abs}\left(y(n) - (a + \mathrm{j}b)\right) \tag{7}$$

$$\theta = \frac{1}{\mathrm{N}} \sum_{n=1}^{\mathrm{N}} \mathrm{angle}\left(y(n) - (a + \mathrm{j}b)\right) + 2\theta_{\upsilon}(n) \tag{8}$$

θ_{υ} is the applied voltage angle at n[th] admittance component

$$y_{11} = \frac{\alpha}{2}\sqrt{\frac{2r^2}{1+\tan^2(\theta)} + a} \quad \alpha = \begin{cases} 1, & -\pi/2 < \theta < \pi/2 \\ -1, & \pi/2 < \theta < 3\pi/2 \end{cases} \tag{9}$$

$$y_{12} = \frac{\alpha}{2}\sqrt{\frac{2r^2}{1+1/\tan^2(\theta)} - b} \quad \alpha = \begin{cases} 1, & 0 < \theta < \pi \\ -1, & \pi < \theta < 2\pi \end{cases} \tag{10}$$

$$y_{22} = 2a - y_{11} \tag{11}$$

$$y_{21} = 2b + y_{12} \tag{12}$$

MEASUREMENT SETUP AND PROCEDURE

To evaluate the limits of the CFAM-model several LED lamps from different manufacturers were tested. All lamps have integrated rectifier circuits and are non-dimmable. Their nominal power varies between 3W and 8W. The measurement of the currents absorbed by the lamps was done with the setup which is shown in Figure 1.

As power source a programmable TESEQ NSG 1007-45 was used. The TESEQ allows measurements at almost ideal sinusoidal voltage with 230V/50Hz as well as any required voltage distortion. For this purpose voltage test conditions are generated in an automated process within MATLAB and sent to power source using a RS-232 serial port and SCPI protocol.

A set of measurements is defined by choosing the highest voltage harmonic, amplitudes of the different voltage harmonics as well as the harmonic phase shift relative to the phase of the fundamental voltage which equals zero. One lamp at a time is connected to the output of the source with a common lamp holder E27. As the behaviour of the lamps turned out to be temperature-sensitive, the lamps are pre-heated under normal operation conditions (230V/50Hz) for about ten minutes.

The data acquisition is made with a Tektronix current probe TCP0030, a Testec TT-SI differential probe and a Tektronix DPO3014 oscilloscope. For analysis of obtained data MATLAB is used. The transformation of the measured voltage and current waveforms from time domain into harmonic domain is done through FFT.

VOLTAGE DEPENDENT CURRENT BEHAVIOUR

All lamps are tested at a base case with only sinusoidal voltage with 50Hz and 230V and with distorted voltages up to the ninth voltage harmonic (450Hz) with varying phase shift between 0 and 360° in steps of 5° as well as varying amplitudes between 1% and 7% of amplitude of the fundamental voltage. The resulting current harmonics are analysed up to order 50 (2.5kHz).

Due to different ballast circuits for rectification and power factor correction each of the tested lamps shows another characteristic current behaviour. While some lamps seem to use simple capacitor smoothed bridge rectifiers, others comprise valley-fill or equivalent circuits. One lamp even absorbs an almost sinusoidal current with high frequency components which is typical for active power factor correction.

All lamps change their current behaviour if the fundamental voltage at the terminals is superimposed with harmonics of different amplitude and frequency. Figure 2 gives examples for two different lamps. While the first lamp for fundamental voltage shows the well-known behaviour of a bridge rectifier with one conduction period per half cycle (a), already at a voltage distortion of 3% 7th harmonic two peaks per cycle appear (b). Also the second lamp is highly non-linear. In case of a voltage distortion

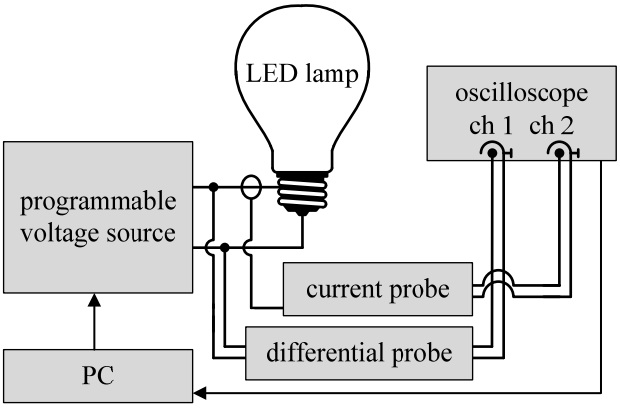

Figure 1: Measurement setup for automated testing

of 9% 7th harmonic (d) the peak value is around 150% compared to fundamental voltage (c), the overall shape and thus the harmonic spectrum differ considerably.

As the shape of currents in time domain vary significantly, also the corresponding loci appear in different forms. For some lamps the loci of harmonic currents have an elliptical shape. Figure 3 depicts the influence of a 3% 5th harmonic voltage distortion on the current harmonics of order three (150Hz) up to nine (450Hz).

At a voltage distortion with 5% 3rd Harmonic the current harmonics of the same lamp follow curves which resemble a half-moon (Figure 4). As shown in the following section, a tensor representation as described in [1] is thus complicated.

Not only the phase of a voltage harmonic is important. Another influence parameter is the amplitude, as plotted in Figure 5. While 3% 3rd harmonic voltage distortion with phase shift 0 to 2π causes the 7th harmonic current to change only in a limited range around the base case, for higher amplitude it follows a circle locus with growing radius.

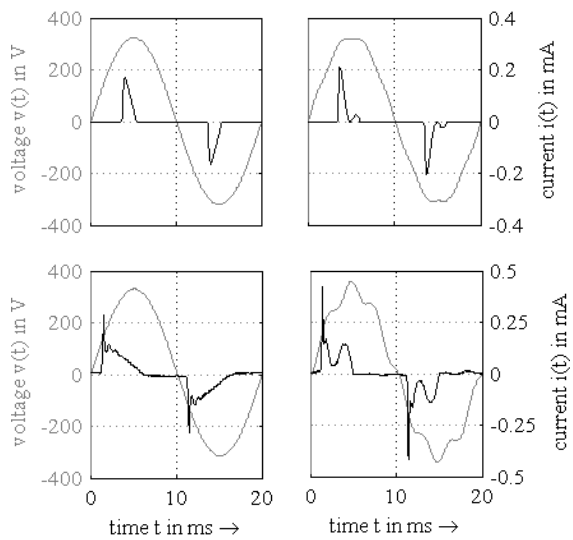

Figure 2: Currents for undisturbed and disturbed voltages

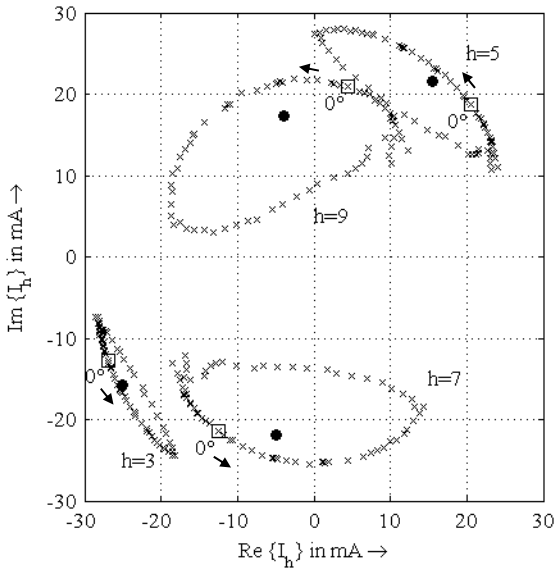

Figure 3: 3rd-9th harmonic current in case of 3% 5th harmonic voltage distortion (0-360°), 5W LED lamp, '•' base case, 'x' measured current, '□' 0° phase shift

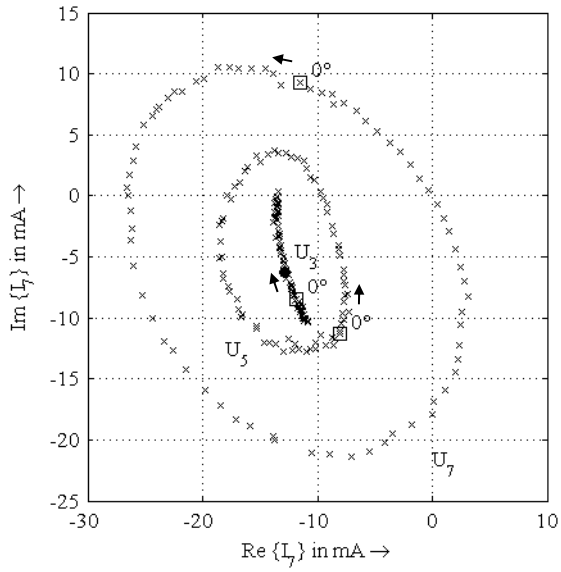

Figure 5: 7th harmonic current in case of 3% 3rd-7th harmonic voltage distortion (0-360°), 8W LED lamp, '•' base case, 'x' measured current, '□' 0° phase shift

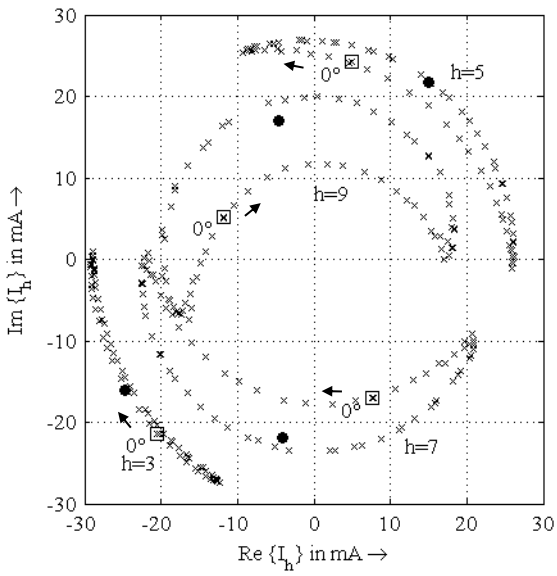

Figure 4: 3rd-9th harmonic current in case of 5% 3rd harmonic voltage distortion (0-360°), 5W LED lamp, '•' base case, 'x' measured current, '□' 0° phase shift

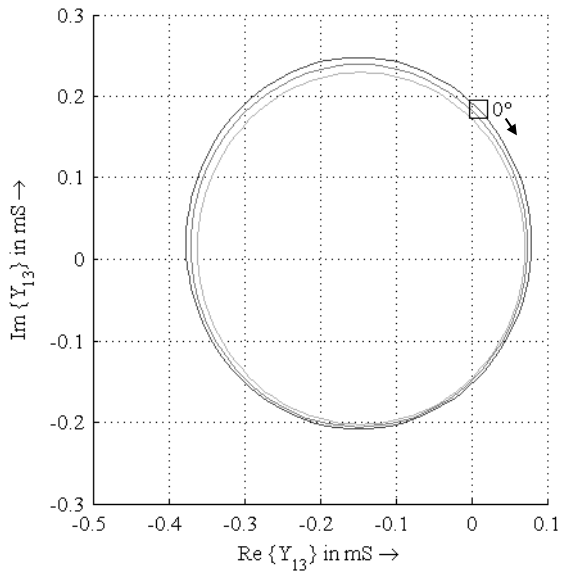

Figure 6: Admittance loci for fundamental current in case of 3%(-), 5%(-) and 7%(-) 3rd harmonic voltage distortion (0-360°), 5W LED lamp, tensor based calculation, '□' 0° phase shift

MODEL PERFORMANCE

If the locus of the harmonic current, as shown in the section before, follows an elliptical shape, the corresponding admittance element can be modelled successfully with the tensor representation. The calculated circles also vary only by few percent if the amplitude of the corresponding voltage harmonic distortion is increased. As example in Figure 6 the admittance loci for fundamental current in case of 3%, 5% and 7% 3rd harmonic voltage distortion is plotted. The loci calculation is based on measurements in steps of 5°. Sufficient results are also received for steps of 10°. Bigger steps result in differences of more than 5%. The best fitting between measurement data and calculated tensor representation is reached for the elements where current and voltage distortion are of the same order (Figure 7).

If the locus of the harmonic current does not follow an elliptical shape, the modelling with tensors is limited as visible in Figure 8. The half-moon-like curves presented in Figure 4 result in loci that cannot be modelled by a simple symmetric circle.

CONCLUSIONS

In this paper the voltage dependent current absorbing behaviour of different LED lamps as an example for small non-linear loads is analysed. It is shown that the current harmonics significantly depend on the phase of a certain voltage harmonic distortion as well as on its amplitude. Based on measurements, the modelling of non-linear loads with frequency coupling matrices is examined. Limits of using tensor representation for single matrix elements are pointed out.

The results show that modelling small loads with high nonlinearity by means of frequency coupling matrices cannot be done easily with reasonable effort and performance.

Future work will have to find extensions of the tensor representation to overcome the difficulties.

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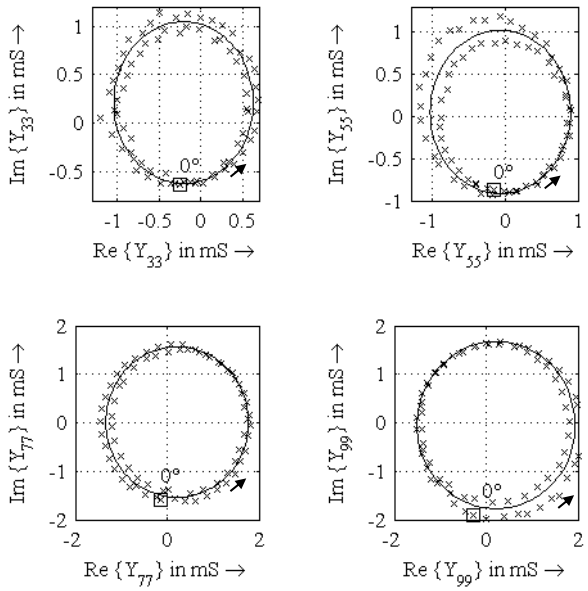

Figure 7: Admittance loci for 3rd, 5th, 7th and 9th harmonic current in case of 3% voltage harmonic distortion of same order (0-360°), 8W LED lamp, 'x' measured current, 'solid circle' calculation based on tensor representation

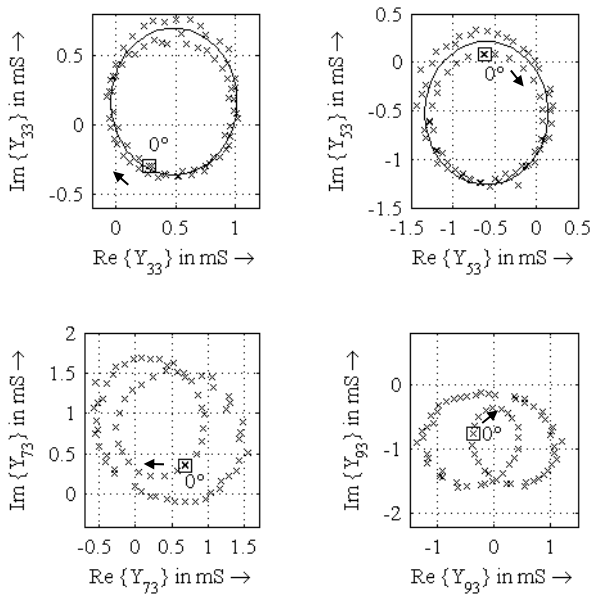

Figure 8: Admittance loci for 3rd, 5th, 7th and 9th harmonic current in case of 5% 3rd voltage harmonic distortion (0-360°), 5W LED lamp, 'x' measured current, '□' 0° phase shift, 'solid circle' calculation based on tensor representation