Risk Analysis Methodologies for Distribution Network Investment Projects and Portfolios

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ABSTRACT

This paper proposes a risk analysis methodology to complement the traditional economical evaluation of investment projects in distribution networks, and proposes a method to assess the overall investment projects portfolio risk.

The first methodology is meant to quantify the risk of projects at a given confidence level with minimum additional evaluation effort from the planning engineer. This requires a specific scenario generation approach to represent the main uncertainty drivers of risk and a sensible mapping of the represented uncertainty into suitable risk measures.

For assessing the risk of a portfolio, this paper proposes to use the Markowitz approximation to make discrete representations of the probability density associated with the value of each project portfolio.

INTRODUCTION

Within the context of investment project analysis, risk is commonly associated with an event or condition that has a negative effect on the objectives of a project or on its profit. Decision makers assess risk identifying variables that might contribute to deviations from the expected rewards or benefits [1].

Typically, the expected profitability of a project is estimated based on operating cash-flows, which result from the provisional account of exploration and represent the difference between benefits and investment costs. Among other possible expected profitability measures, net present value (NPV) is the most popular measure [2]. Both investment costs and corresponding future benefits of such investments are uncertain. In the distribution network context, benefits are usually strongly related on future demand, whose growth rate needs to be estimated under uncertainty. Investment costs are also usually uncertain as network investments are complex and subject to environmental and governmental constraints.

Quantile based risk measures of economic indicators rely upon the characterization of the uncertainty of benefits and costs of the investment project.

In this paper, we propose to use historical data on both load growth and investment cost expectations, together with data on actual demand growth and incurred costs, to characterize forecast errors. Based on errors uncertainty, we then present a procedure that generates future error scenarios to find the risk values of the load growth rate and investment cost to then be able to estimate the lowest value of an investment project, i.e., the net present value at risk, NPVaR.

Based on NPV and NPVaR values we represent a simplified NPV probability distribution functions for each project and propose a Markowitz approximation to assess the portfolio risk [3].

RISK ANALYSIS OF AN INVESTMENT PROJECT

Traditionally, investment projects in distribution networks are prioritized based on their expected profitability as measured by NPV. To calculate the NPV, a set of predictions about demand growth and investment amounts are central because they are drivers of expected benefits and costs, respectively. If we have information on the probability distributions of such drivers, we may calculate a joint probability distribution of the variables and obtain the risk values for a desired confidence level.

In this section we present the methodology proposed to characterize uncertainty, generate representative scenarios for such uncertainty and explain how to compute NPVaR based on the set of generated scenarios.

Uncertainty Characterization

Uncertainty in demand growth rate can be studied by looking into the historic distribution of the annual demand growth forecast errors, ε_a. Investment costs uncertainty can also be studied by looking into the distribution of the investment cost forecast errors, ε_i.

Let us define the demand forecast error as,

$$\varepsilon_D = \frac{D_a - D_f}{D_a} \qquad (1)$$

where D_f is the forecast value in year t for the demand in year $t+N$ and D_a is the actual demand value in year $t+N$. The annual demand forecast error is the geometric mean of the cumulated error, i.e., $\varepsilon_a = \sqrt[N]{1+\varepsilon_{D_N}} - 1$, where ε_{D_N} is the cumulated error after N years.

Similarly, investment cost forecast errors can be defined as,

$$\varepsilon_i = \frac{I_a - I_f}{I_a} \qquad (2)$$

where I_f is the forecast value of investment cost and I_a is the actual value of the investment cost.

Examples of properties of the demand and investment cost forecast errors for a particular distribution network area are shown in Table 1. Note that there is a risk of loss if the actual investment cost is higher than the forecast investment cost and if the actual growth rate is lower than the forecast, so for generating scenarios for ε_i, ε_a and their joint probability distribution function, we must negate ε_i.

Table 1 Central Moments for the Two Error Variables

	Expected Value (%)	*Standard Desviation (%)*	*Skewness*	*Kurtosis*
$\boldsymbol{\varepsilon_a}$ [pu]	0.00	3.10	-0.71	6.69
$\boldsymbol{\varepsilon_i}$ [pu]	-7.70	19.60	0.06	2.64

Getting Net Present Value at Risk

An investment project in distribution networks is evaluated by its economic indicators as measured by NPV, which is given by the difference between the discounted operational benefits and the discounted investment costs, at a given discount rate *t*. Quantifiable operational benefits depend on the project purpose but are typically chosen to quantify the reduction of energy losses and the reduction of the number and duration of power interruptions [4], [5], usually measured as energy not supplied (ENS). In (3) we give a possible NPV analytical expression that includes both energy losses and ENS reduction benefits over a planning horizon of H years.

$$\mathrm{NPV} = -I_f + \sum_{k=1}^{H} \frac{1}{(1+t)^k} \times \left(B_{Loss}(\alpha_f, k) + B_{ENS}(\alpha_f, k)\right) \tag{3}$$

where α_f and I_f are the forecast (expected) demand growth rate and investment cost, respectively.

The benefits from loss and ENS reductions, $B_{Loss}(\alpha_f, k)$ and $B_{ENS}(\alpha_f, k)$, are nonlinear functions of load and require network analysis to be reevaluated under different load conditions. Therefore, the NPV probabilistic distribution function cannot be determined analytically by repeatedly computing NPVs for each and every of the error scenarios, which must be many in order to represent the distribution tail adequately.

To avoid such overburden, we propose to determine the maximum possible errors instead of the lowest possible NPV, for a given confidence level, and then approximate the lowest possible NPV by the NPV obtained for the maximum possible errors –NPVaR. To determine maximum errors, one has to generate the error scenarios in a way that errors become ordered. As NPV is a decreasing function of investment cost and an increasing function of benefits that are also increasing functions of demand growth, we have to guarantee that error scenarios provide monotonically increasing negative investment cost errors and monotonically increasing demand growth errors.

The maximum error values at a confidence level of $(1 - P^*)$, ε_a^* and ε_i^* can then be obtained with the joint probability distribution function of the forecast errors (4) and be used to compute an approximation of net present value at risk, NPVaR, with (5)

$$\varepsilon_a^*, \varepsilon_i^*: \sum_{-\infty,-\infty}^{\varepsilon_a^*,\varepsilon_i^*} f_{\varepsilon_a \varepsilon_i}(\varepsilon_a, \varepsilon_i) = P^* \Leftrightarrow \\ \Leftrightarrow F_{\varepsilon_a \varepsilon_i}(\varepsilon_a \le \varepsilon_a^*, \varepsilon_i \le \varepsilon_i^*) = P^* \tag{4}$$

$$\mathrm{NVPVaR} = -\frac{I_f}{1-\varepsilon_i^*} + \sum_{k=1}^{H} \frac{1}{(1+t)^k} \times \\ \times \left(B_{Loss}\left(\frac{\alpha_f + \varepsilon_a^*}{1-\varepsilon_a^*}, k\right) + B_{ENS}\left(\frac{\alpha_f + \varepsilon_a^*}{1-\varepsilon_a^*}, k\right)\right) \tag{5}$$

The Scenario Generation Method

In decision making under uncertainty, one must represent uncertainty in a form suitable for quantitative analysis. If uncertainty is expressed in terms of multivariate continuous distributions or a discrete distribution of many outcomes, one normally faces two possibilities: either (*i*) creating a decision model with internal sampling or (*ii*) finding a simple discrete approximation (set of scenarios) of the given distribution that serves as input to the model [6],[7].

An ideal situation would be that of the set of scenarios being such that it represents the whole universe of possible outcomes of the random variables [8]. To determine representative pessimistic values at the required confidence level it is necessary to generate scenarios while maintaining the properties of uncertainty, as characterized. In the following we formulate the optimal scenario generation problem introduced by Høyland and Wallace [6] applied to generate scenarios for the two independent random variables considered: annual demand forecast error, ε_a, and investment cost forecast error, ε_i.

Notation

n	Number of random variables ($n = 2$).
S	Number of scenarios.
ε	Scenario vector of size $n \times S$.
p	Probability vector of size *S*.
m_{nv}	Central moment *v* for the statistical property of *n*.
M_{nv}	Specified value of the central moment *v* of *n*.
c	Covariance.
w_v	Weight of statistical property *v* in S.

Formulation

Høyland and Wallace present a method based on non-linear programming to generate a limited number of scenarios that satisfy specified statistical properties of uncertainty. The basic idea underlying the scenario

generation is to minimize the distance between the statistical properties of the generated scenarios and specific properties of uncertainty, as characterized. The purpose is to weight the sum of quadratic distances between different properties of uncertainty.
The scenario generation procedure presented before is here applied to generate scenarios for the two independent random variables considered: annual demand forecast error, ε_a and investment cost forecast error, ε_i. The statistical properties considered are the four central moments: expected value, standard deviation, skewness and kurtosis, and the covariance.
A set of S terms $(\varepsilon_{aj}, \varepsilon_{ij}, p_j)$, where p is a probability, is determined when solving a non-linear optimization problem with (6) for null covariance and with four constraints which define the distribution and ensure that the two error distributions generated are monotonic.

$$\min_{\varepsilon_a,\varepsilon_i,p} \sum_{n=1}^{2}\sum_{v=1}^{4} w_{nv}(m_{nv}-M_{nv})^2 + c^2$$

$$\text{s.t} \quad \sum_{j=1}^{S} p_j = 1 \qquad \sum_{j=1}^{S-1} \varepsilon_{aj} \le \varepsilon_{aj+1} \qquad \sum_{j=1}^{S-1} \varepsilon_{ij} \le \varepsilon_{ij+1} \qquad p_j \ge 0 \tag{6}$$

RISK ANALYSIS OF A PORTFOLIO OF INVESTMENT PROJECTS

Harry Markowitz introduced the modern portfolio theory where explains how to select a set of projects in order to maximize the portfolio expected return for a given amount of portfolio risk. The Markowitz model defines the necessary information to choose the best portfolio to any risk levels are contained in two statistical parameters: expected value and covariance [3].
For assessing the risk of a portfolio, we use the Markowitz formula to make discrete representations of the probability distribution function associated with the value of each project portfolio.
The value of distribution network investment projects is represented by three Diracs with symmetry in relation to its expected NPV (as shown in Figure 1): NPVaR, obtained with the methodology introduced previously, with probability of $1-g$, NPV of the project with probability $2g-1$ and $2NPV-NPVaR$ with probability $1-g$, where g is the guarantee level intended, $g=1-P^*$.

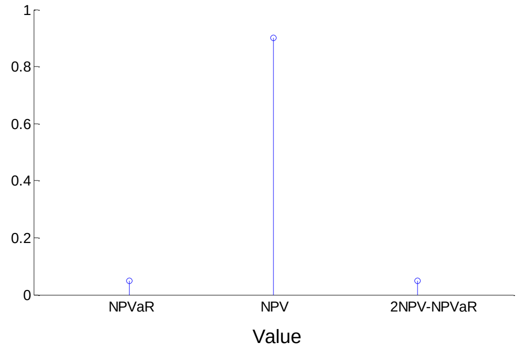

Figure 1 Distribution network investment project representation with g=95%

Modern portfolio theory models an asset's return as a normally distributed function, so we represent the investment projects as a normal distribution function with an expected value and a standard deviation given by (7) and (8).

$$\mu_i = NPV \tag{7}$$

$$\sigma_i = \sqrt{\sum_{n=1}^{3}(x_n-\mu_i)^2 f_n} = \sqrt{2(\mathrm{NPV}-\mathrm{NPVaR})^2 \times P^*} \tag{8}$$

The expected value and covariance of the portfolio obtained by the Markowitz formula are given by (9) and (10) respectively, where σ_i is the standard deviation of the project i and σ_{ij} is the covariance between the project i and j, that is zero for distribution network projects, due to investment costs be independent between projects and in the growth demand uncertainty have been isolated the dependence conjectural.

$$E\{p\} = \sum_{i=1}^{n} NPV_i \tag{9}$$

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \sigma_i^2} \text{ for } \sigma_{ij} = 0\ \forall ij \tag{10}$$

The $NPVaR$ of the portfolio with n projects can be obtained by the Markowitz formula by (11), and the portfolio risk is then given by (12), which represent the value that might be lost with a probability of P^*.

$$\mathrm{NPVaR_p} = \mathrm{NPV_p} - 1{,}644\sigma_p \tag{11}$$

$$risk = \frac{\mathrm{NVP_p\text{-}NPVaR_p}}{\mathrm{NPV_p}} \tag{12}$$

NUMERICAL RESULTS

In this section we present the results of the methodologies introduced previously. We generate a set of scenarios to assess the risk values for an investment project at a given confidence level to the decision makers to be able to get NPVaR to complement the traditional economical evaluation of investment projects in distribution networks. We then assess the portfolio risk with the Markowitz approximation and the convolutions of the projects.

Risk Analysis of an Investment Project

We obtain the forecast errors with the scenario generation method proposed previously in order to get the monotonically increasing function of NPV in order to be able to estimate NPVaR with (5).

Figure 2 shows the joint cumulative distribution function, $F_{\varepsilon_a\varepsilon_i}(\varepsilon_a, \varepsilon_i)$, those results are obtained with the scenario generation method (6) for *S=27*. A perfect match with the specific properties of Table 1 and zero covariance is now obtained when the number of scenarios is ten or higher.

$(\varepsilon_a^*, \varepsilon_i^*)$ at a confidence level of 95% is shown by a red dot and at a confidence level of 90% by a green dot. In Table 2 we show ε_a^* and ε_i^* on the size of *S*.

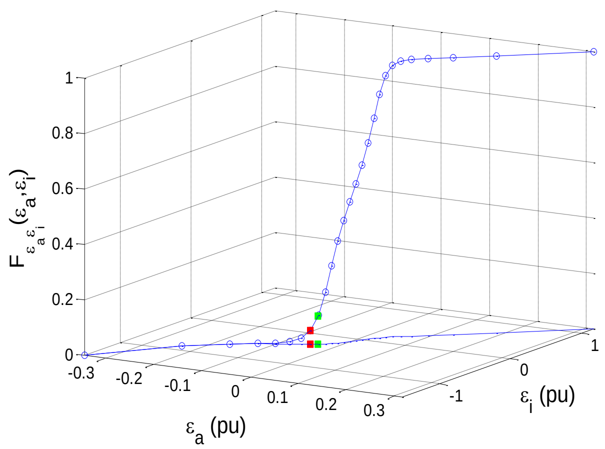

Figure 2 Results for the joint cumulative distribution function, $\boldsymbol{F_{\varepsilon_a\varepsilon_i}(\varepsilon_a, \varepsilon_i)}$, and the risk values of $(\boldsymbol{\varepsilon_a^*, \varepsilon_i^*})$ at the 95% and 90% confidence levels

Table 2 Results from (6) for $\boldsymbol{\varepsilon_a^*}$, $\boldsymbol{\varepsilon_i^*}$ for Different S at a 95% Confidence Level.

S	ε_a^*	ε_i^*
10	-0,06	0,36
14	-0,05	0,35
27	-0,04	0,33
33	-0,04	0,32
50	-0,04	0,33

Through Table 2 we can conclude that for *27* scenarios or higher the results are mostly alike, therefore in Table 3 we present the risk values ε_a^* and ε_i^* with a 5% and 10% guarantee obtained for *S= 27*.

Table 3 Risk values with a 5% and 10% guarantee level for *S=27*

	5%	*10%*
ε_a^*	-0,04	-0,03
ε_i^*	0,33	0,29

With this result, we are able to calculate the approximation re-evaluate, for the maximum error values of Table 3. This represents a major advantage over the standard risk approach for which the planner would have to (*i*) re-evaluate the investment cost and its benefits for dozens of possible scenarios, (*ii*) compute the NPV for each of the scenarios, (*iii*) order the scenarios from the lowest to the highest NPV, to then be able to (*iv*) sum their probabilities until they yield the P^* to finally (*v*) determine the minimum NPV.

Risk Analysis of a Portfolio of Investment Projects

In this section we validate the Markowitz formula approach to get to the portfolio risk. We compare the risk results achieved with the methodology defined with the actual probabilistic distribution of a portfolio.

The actual distribution of a portfolio is given by the convolution between the distribution function of the portfolio projects, $f_{X+Y}(z) = f_x(x) * f_y(y)$, namely, the convolutions of the Diracs represented in Figure 1.

The central limit theorem states that, given certain conditions, the arithmetic mean of a sufficiently large number of iterates of independent random variables, each with a well-defined expected value and well-defined variance, will be approximately normally distributed, regardless of the underlying distribution. For this reason, the Markowitz formula should only be used when the portfolio include many projects.

With the equation (12) we determine the portfolio risk of the Markowitz formula and the portfolio risk of the actual distribution with the convolutions. In Figure 3 we present the outcomes of this comparison for different portfolios established by projects with different NPV.

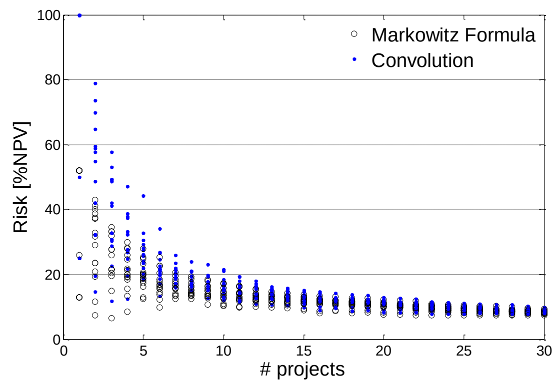

Figure 3 Risk evolution with the number of independent projects

The black dots represent the risk evolution of a portfolio with the number of projects of this portfolio achieved by the Markowitz formula. And the blue squares represent the same evolution but for the portfolio actual distribution.

We conclude as expected that the Markowitz formula is only a good approximation when the portfolio include many projects.

As predictable, the portfolio risk decreases with the size of the portfolio, in fact, for a large number of projects, the risk becomes practically independent of the portfolio.

CONCLUSION

This paper proposes two risk analysis methodologies that rely on a modified NPV analysis of investment projects in distribution networks: *(i)* risk analysis for an investment project and *(ii)* risk analysis for a portfolio of investment projects.

The first methodology is used to determine the lowest possible project value by evaluating the project for the worst possible outcomes of the uncertainty drivers: the highest possible investment cost and the lowest possible load growth. Such outcomes can be determined by solving a non-linear optimization problem to generate a set of scenarios that generates monotonically increasing forecast errors for the load growth and investment cost (negated). We illustrate the application of the methodology in the specific context of the distribution projects evaluation and highlight its advantages.

The second methodology is used to determine the approximated portfolio risk based on simplified probability distributions of individual project NPV values.

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