APPLICATION OF VARIANCE-BASED SENSITIVITY ANALYSIS TO ISSUES OF STABILITY AND PROTECTION IN DISTRIBUTION GRIDS – TWO CASE STUDIES

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ABSTRACT

This paper addresses the field of analysis and evaluation of electrical models with high-dimensional parameter spaces. Two case studies within the context of stability and protection of distribution grids demonstrate the usability of sensitivity analysis (SA) as an efficient assessment tool for practical applications whilst comparing the different methods with respect to the resulting computational time and accuracy. An analysis identifies the SA method that can be used best for different modelling purposes.

INTRODUCTION

The increasing number of distributed generation (DG) throughout Europe leads to a change in the load and infeed behaviour of energy systems, sometimes making more complex models necessary in order to accurately describe their changing technical characteristics. In order to handle high numbers of input parameters, qualitative and quantitative methods of global sensitivity analysis (GSA) can be used to comprehensively estimate the effect of input parameters of computational models onto their outputs.

Within this paper GSA methods representing the current state of research are described and compared with focus on their run time and their solution's accuracy. Two case studies within the context of stability and protection in distribution grids show their practical usability.

The following nomenclature for the explanations of the GSA is used.

$E(\dots\mid\dots)$	conditional expectation
EE_i	elementary effect of the input parameter i
M	number of calls of the model analysed
N	sample size
X	$N \times k$ sample matrix
X'	$N \times k$ sample matrix without a change in the ith column
k	number of input parameters with $\{1, \dots, i\text{-}1, i, i{+}1, \dots, k\}$
$p(\dots)$	density function
s	number of search-curves
s_i	first order sensitivity index of the input parameter i
s_{Ti}	total effect sensitivity index
x_i	input parameter i
y	output parameter
σ^2	variance
Δi	increment in the input's domain

METHODOLOGY

According to [1] SA aims to quantify the relationship between a model's input and output parameters. It can be differentiated between local (restricted to one input parameter) and global as well as qualitative and quantitative methods. Qualitative methods (e.g. the methods based on the analysis proposed by Morris [2]) are computationally less costly than quantitative methods (e.g. the method firstly proposed by Sobol [3]). Quantitative SA, on the other hand, is able to provide an analytically comprehensive analysis of the model as the approximation error is calculated in addition to the sensitivity indices [4].

Due to the non-linear and high-dimensional problems analysed within this paper, the authors focus on global methods. The latest enhancements in each method are implemented: The radial procedure by Morris is used as a qualitative analysis [5]. The extended Fourier Amplitude Sensitivity Test (eFAST) [1], which is an improvement of the original method proposed in [6], and the method of Sobol using the enhanced estimator proposed by [7] are used for a quantitative, variance-based analysis. Other global methods, such as regression and correlation analysis, are not regarded within this paper as the model's linearity is not given for the models assessed.

Quantitative sensitivity analysis

Figure 1 illustrates the procedure of variance-based SA. The method uses the known variance of the model's input parameters to compute the unknown variance of the output parameters by simultaneously changing all input parameters at a time using stochastic variables. Afterwards, the conditional variance of the output parameters is calculated by a predetermined estimator, thus leading to the sensitivity indices as defined below.

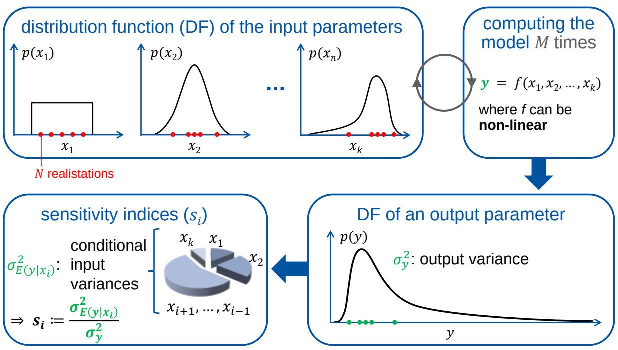

Figure 1: General approach of variance-based SA

Following the common definitions of the conditional expectation $E(y|x_i)$, the output's variance σ_y^2 and the conditional variance $\sigma_{E(y|x_i)}^2$ of a density function $p(x_i)$ with the input parameter x_i

$$E(y|x_i) = \int_{-\infty}^{\infty} y \cdot p(y|x_i) dy \quad (1)$$

$$\sigma_y^2 = E[(y - E(y))^2] \quad (2)$$

$$\sigma_{E(y|x_i)}^2 = E\left[\left(y - E(y|x_i)\right)^2 | x = x_i\right] \quad (3)$$

the first order sensitivity indices s_i for the output parameter y can be defined as follows [7]:

$$s_i = \frac{\sigma_{E(y|x_i)}^2}{\sigma_y^2} \quad (4)$$

Therefore, the first order sensitivity indices represent the conditional and normalized influence of the variance of the input parameter x_i on the variance of the output y. The total effect sensitivity indices s_{Ti}, on the other hand, represent the first order effects plus the interdependent interactions of the input parameter x_i with all the other input parameters:

$$s_{Ti} = 1 - \frac{\sigma_{E(y|\sim x_i)}^2}{\sigma_y^2} \quad (5)$$

Using a quasi-random Sobol sequence [8] the sample matrices X and X' are created in order to calculate s_i and s_{Ti} according to the procedure described in [7]. The following transformation to the Fourier domain for the eFAST method was used [10]:

$$x_i = \frac{1}{2} + \frac{1}{\pi} \arcsin(\sin \omega_i s) \quad (6)$$

Qualitative sensitivity analysis

In order to get equivalent results for both, quantitative and qualitative GSA, the following measures are proposed in [5] for the so-called radial method by Morris based on the elementary effect EE_i:

$$EE_i = \frac{y(x_1, \dots, x_i + \Delta i, x_k) - y(x_1, \dots, x_k)}{\Delta i} \quad (7)$$

$$s_i^{Morris} = \frac{\sum_{j=1}^{N} |EE_{i,j}|}{N} \quad (8)$$

$$s_{Ti}^{Morris} = \frac{\sum_{j=1}^{N} \left(EE_{i,j} - s_i^{Morris}\right)^2}{N} \quad (9)$$

In contrast to the original method by Morris, the radial procedure chooses the increments Δi in such a way that the whole parameter space is regarded within the analysis.

COMPARISON OF THE GSA METHODS

Run time

The three methods' numbers of calls of the model to be analysed can be quantified as follows for estimating s_{Ti}:

- radial Morris: $M_{Morris} = N \cdot (k + 1)$
- eFAST: $M_{eFAST} = N \cdot k \cdot s$
- Sobol: $M_{Sobol} = N \cdot (2^k - 1)$

Figure 2 exemplarily illustrates these "run times". Due to the exponential behaviour of Sobol and keeping in mind the sample size (N) and number of search curves (s), this method should only be used for small numbers of input parameters ($k \lessapprox 6$).

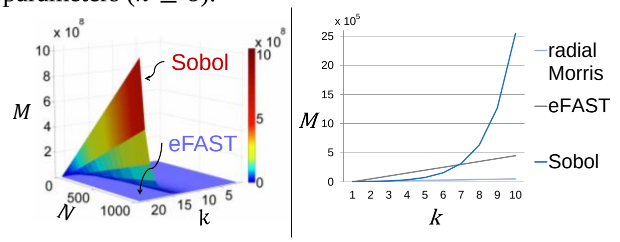

Figure 2: Number of model calls for Sobol and eFAST (left) and for all three methods (right with $\boldsymbol{N = 500}$) using $\boldsymbol{s = 10}$

Accuracy

The difference of the solution's accuracy of the quantitative methods is negligible: Figure 3 exemplarily shows the results of the analytically solvable ("calculable") *g-function* [7], which represents the overall conclusion that the Sobol and eFAST method produce – apart from minor differences – identical results.

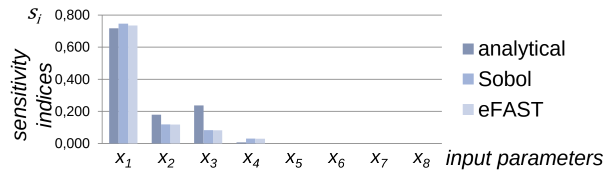

Figure 3: Example comparison between the analytical ("calculable") solution and the solution of the quantitative methods for Sobol's *g-function*

Generally, the solutions of the quantitative and qualitative results are comparable [5], it should always be kept in mind that Morris might create estimation errors when the correctness of the model analysed is not known. Figure 4 exemplarily demonstrates the difference in the analysis: It shows the evaluation of a symmetric fault (3-phase) at the cable's end of a single line low voltage feeder (NYY-J 4x150). The variation of the cable length (0.001 to 1,000 km) as well as the transformer's apparent power (50 to 630 kVA) and impedance respectively onto the subtransient short-circuit AC current is analysed. The current is calculated using the method of the equivalent voltage source [11].

This method intuitively produces correct results from the technical point of view for realistic cable lengths up to about 2 km: The nearer the fault is located to the transformer, the higher is the influence of its reactance and the lower is the influence of the cable's total impedance. However, when the upper bound of the cable length is increased to unrealistically high values the calculation model becomes physically incorrect. While the quantitative SA (eFAST) is able to detect the incorrect model making the influence of the estimation error higher, the qualitative SA (radial Morris) leads to wrong estimation of the sensitivity indices.

These conclusions are only valid as long as the method applied converges. Within this paper the authors assume a convergence of the method as long as there are no changes in the order of the sensitivity indices for an increase in time of 10% of the total computational time of the GSA.

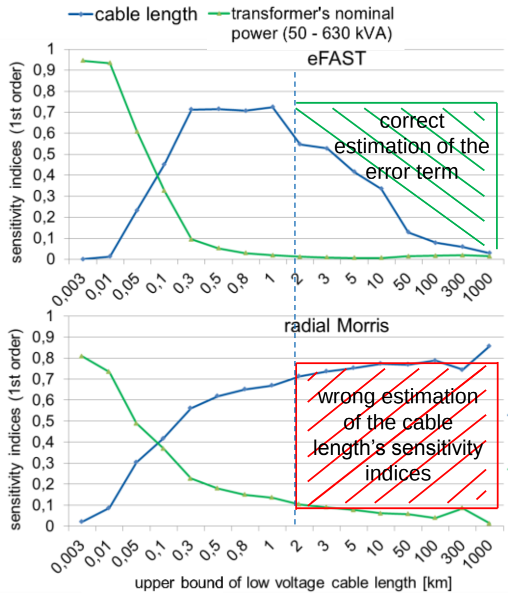

Figure 4: Comparison between the solutions' accuracy of a quantitative (top: eFAST) and qualitative (bottom: radial Morris) method

CASE STUDIES

Parameterization of dynamic models of synchronous generators

Within the first case study it is shown that GSA can be used to fit dynamic models such as the *Standard pu* model of *SimPowerSystems*™ of synchronous generators (70, 250 and 365 kVA) based on the standard time domain model (e.g. [11]). The models are fitted with regard to the impedances (x_d'', x_d', x_d, x_q'', x_q', x_q) and time constants (t_d'', t_d', t_q'', t_q') of the model. These models are used for stability analysis. The model's benchmarks are measurements of generation units in a low voltage ride through (LVRT) [9] testing environment using voltage drops down to 30, 50 and 75%, varying fault durations of 150 to 900 ms and varying loads of 50 to 100%. With the help of the sensitivity analysis it shall be estimated which of the parameters (reactance and time constants) are most important and which overall parameterization fits best for all LVRT measurements with regard to the subtransient short-circuit AC current.

Due to the high computational time of the model including the LVRT setup, an approach combining qualitative and quantitative SA methods shown in Figure 5 is used: Firstly, the radial Morris method is applied to identify the most important inputs. Morris can be applied due to the fact that the model used can be regarded as correct and complete [11]. Secondly, Sobol sequences are used as an efficient sample generator to estimate the best fitting parameterization for the most important inputs that could be identified (see Figure 6: x_d'', x_d', x_d, x_q'').

The GSA could successfully be used as an identifying tool in the first and Sobol sequences as an efficient sample generator in the second step. Due to the results, it is assumed that the smaller the nominal power of the generator is, the higher is the influence of the quadrature-axis subtransient reactance x_q''. This is the subject of the current research.

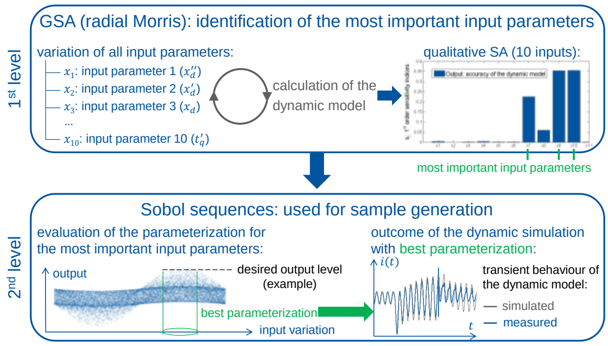

Figure 5: Two-level approach for applying GSA to models with a high computational time (example: parameterization of a synchronous machine)

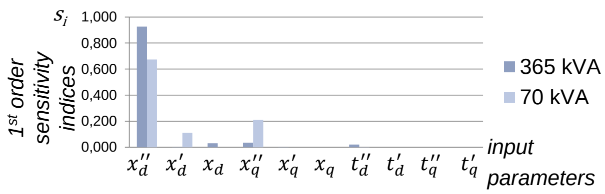

Figure 6: Results for the 1st level (Morris) analysis exemplarily showing the 365 kVA and 70 kVA synchronous generators

Steady-state short-circuit calculations

The second example is the application of GSA to steady-state short-circuit calculations under high DG penetration. DGs increase the system complexity by their interaction in fault cases. The example shows how GSA may systematically aid in generating knowledge on the influencing factors in new complex grid situations.

The distribution system (DS) under study is given in Figure 7. The DS is highly penetrated with inverter interfaced distributed generators (IIDG).

In case of grid faults IIDG behave as controlled current sources unlike the overlaying grid or synchronous generators. Numerous degrees of freedom exist during the design of the IIDG's control, which are typically not known in detail for practical purpose studies [12]. Two methods for short-circuit calculation that enable considering the IIDG fault behavior have been developed and cross-validated in [12]. The cross-validation is performed for identical parametrization of the grid, IIDG sizing and IIDG control design models and parameterization. The two methods yield results for relay current amplitudes (CT 1-3) in case of three phase faults at N5 and N16 that match within 0.02 pu and 0.005 pu.

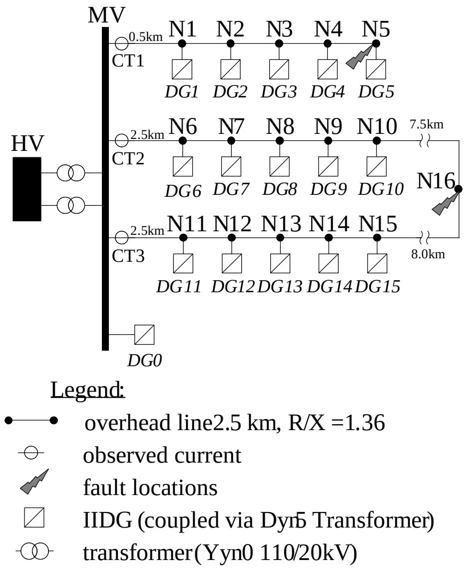

Figure 7: Study grid (details are given in [12])

In order to benchmark this deviation, deviations from the determined CT 1-3 values due to a variation of the IIDG's control shall be determined. The hypothesis to be verified is that variation and diversity (both unknowns in practical studies) of IIDG control in a DS is significantly more influential to the CT currents than an error due to the choice of method. It shall be determined which control aspects are of dominant influence.
A multi-step approach with increasing complexity and number of input parameters is used:

1. Identical variation of all IIDG controls
2. Grouping of IIDG and individual variation of control per group in order to reflect diversity
3. Additional variation of pre-fault reactive power

The variations performed on the IIDG control have the parameters given in Table 1 (details are given in [12]):

Table 1: IIDG control parameters varied in GSA

parameter	symb.	values
characteristic of required grid supporting reactive current injection ΔI_B	Shape	• SDLWindV • TC2007
definition of reference voltage U_0	U0	• nominal volt. • prefault voltage
residual voltage U_1 measurement procedure	U1	• one-time initial • continuous upd.
definition of reactive current offset for ΔI_B	IB0	• zero • prefault
maximization of active current until available active power or current overrating limit is reached	IW	• yes • no

The variation of the pre-fault situation per IIDG group is affecting the reactive power injection (QPre):

- active power injection only
- power factor 0.95 (overexcited at busbar, underexcited otherwise)

These 6 input parameters (Shape, U0, U1, IB0, IW and QPre) are varied in three groups of IIDG newly assigned per fault point:

- connected to busbar
- connected to healthy feeders
- connected to faulty feeders

In total a number of 18 binary value input parameters are varied in the SA. Due to the high number of input parameters, the radial Morris approach is chosen as the SA method. A number of $19{,}000 \ll 2^{18}$ calculations are performed per fault point. Figure 8 shows the first order sensitivity indices for current amplitudes in the faulty feeder's CT. The convergence of the GSA has been positively verified.

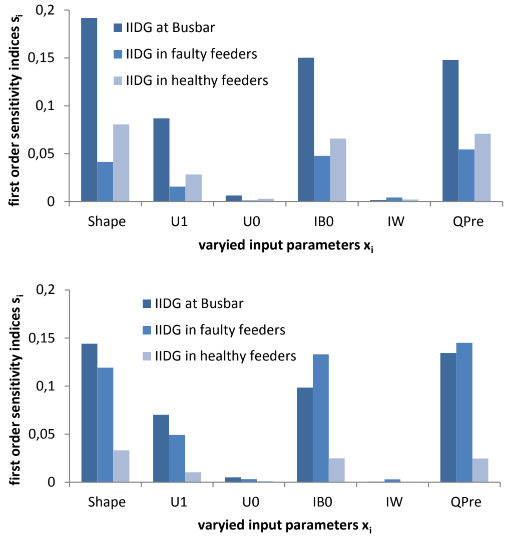

Figure 8: First order sensitivity indices of CT currents of faulty feeders for three-phase faults at N5 (top) and N16 (bottom)

As all groups of IIDG for both fault points have non-negligible influence, an interaction of all IIDG in the DS becomes obvious. Three out of the five varied control parameters (Shape, U1, IB0) have significant influence in both cases. In both cases the pre-fault reactive power injection QPre is of influence. For identical voltage sags these parameters affect the reactive current injection, which in turn influences amplitudes and especially relative phase angles of the IIDG injections. Whilst the radial Morris method does not allow for quantitative comparison due to its qualitative character, the different heights of influence per group per parameter and per case reflect

- the IIDG rated powers aggregated per group and
- the distance of the fault point to the busbar affecting the voltage sag depth at the busbar and in the healthy feeders.

The negligible influence of the parameter IW is ascribed to the pre-fault situation of maximum active power injection, which results in small changes of the active current only. Other pre-fault situations may lead to an increased influence.

The negligible influence of the parameter IB0 is ascribed to the relatively small admissible voltage bandwidth of ±10% of nominal voltage.
The analysis shows that in the relatively large set of influential parameters no outstanding parameter can be identified.
The resulting deviations of the absolute current amplitudes in the CT of the faulty feeder in comparison to the cross-validated cases are shown in Figure 9.

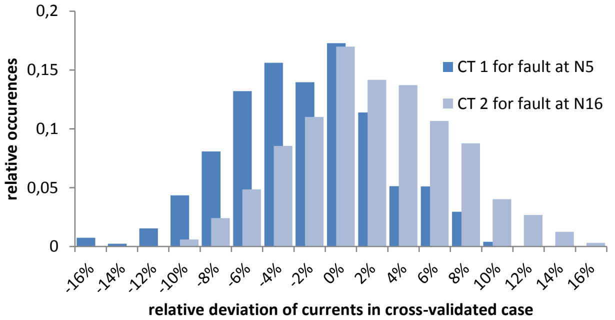

Figure 9: Empirical distribution of relative deviations of CT currents from cross-validated values of Type A method in [12]

Comparing these relative deviations due to the variety and diversity of IIDG control parameters with the relative error due to the choice of the method calculation (see [12]), the former must be attributed as significant.
The application of the GSA to the study case created valuable knowledge on the influencing factors on the short-circuit currents observed at relay positions in DS with high IIDG penetration, that a one at a time parameter variation would not have yielded.

CONCLUSION AND OUTLOOK

It could be shown that global sensitivity analysis is a comprehensive tool to analyse models with a high-dimensional parameter space within the context of electrical power engineering. The vast variety of the practical usability and possible applications could be shown for two case studies: The GSA methods could successfully be applied to problems of dynamic simulation as well as to topics in the context of steady-state analysis. It was shown in which cases the application of the qualitative analysis (Morris) is valid and when to use either Sobol or eFAST with regard to their computational run time if a quantitative analysis is necessary. With the help of GSA new research topics could be identified and existing presumptions could be proved in a quantitative way.
The following topics are in the focus of further research:

- Setup of a testing environment for small-scaled synchronous generators (< 70 kVA: current minimum of generators tested) within the testing centre of the Institute for High Voltage Technology, RWTH Aachen University
- Analysis of the steady-state models of these generators and possible identification of a functional relationship between the nominal power and the influence of the quadrature-axis subtransient reactance
- Systematic consideration of IIDG control parameters in steady-state short circuit calculations, suitable for practical purpose studies

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