EVALUATING THE COSTS OF LOAD SHEDDING SERVICES FOR ELECTRICITY NETWORKS

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ABSTRACT
The aim of this paper is to present a model evaluating the price of a contract between an aggregator and a balance responsible (BR) entity. The contract terms allow the BR to optionally directly control several loads at the end-user’s home where smart equipment, called energy boxes, are installed. These devices give access to control certain non-vital loads for load shedding purposes. The model of this paper is developed from the perspective of the BR which seeks to minimize the balancing cost. This model addresses a new evaluation approach of load shedding via real option theory. A case study for residential load shedding services at a city scale will be described. The results of the proposed method lead to a threshold value after which the contract is considered to be unprofitable for the BR.

INTRODUCTION
Electricity market deregulation has taken place in many countries. It has changed the organization of the electrical system and has modified the way the electrical system used to operate. With it, new players have appeared on the market competing and completing each other’s objectives such as: independent producers, suppliers, local balance responsible, aggregators etc.. However this new pattern creates new services encouraging the end user to be an active player responding to urgent needs and get compensated upon requests. Among these services load shedding interests this paper.

General Context
This paper focuses on two independent players, an aggregator and local balancing responsible (BR). The aggregator in this paper is the player standing behind load shedding services; it installs needed equipment and sells load shedding services to other players as needed. The uncertainties surrounding demand and generation forecasts and contingencies can result in what are known as imbalances [1]. The BR is the actor taking the responsibility of ensuring the balance of the grid [1]. In this paper the BR uses multiple generation units which do not have the same cost of energy production (€/MWh), nor power production capacity. As a matter of fact using extra generation units for peak demand hours on the grid may increase the price of electricity, ending up with higher electricity bills, which may be unsuitable for the end users [2][3]. This may drive end users to accept having energy boxes (smart equipment allowing load shedding services) in return to some discount on their electricity bills.

Figure 1 summarizes the scope of this article, the users participating in load shedding program are shown in red circles, the electrical demand profile (ED) is presented by the blue curve and the different methods of generation are shown on the left.

The considered network has a nominal power $P_n$, and maximal power $P_m$ expressed in MW. The equipped users constitutes $\alpha_i$ % of the total end users. The controllable loads are limited to electrical heating systems.

Objectives
This paper tries to evaluate the cost of the load shedding services proposed by the aggregator from the perspective of BR using real option theory. This service gives the BR access to energy boxes installed at the end users by the aggregator through individual contracts permitting load shedding. As a result the BR can maintain the balance of the grid during peak demand by modifying the load profile. Such load shedding techniques can be used to obtain less expensive balancing cost. In other words, without the need of using additional production units that may be extremely expensive when compared to standard production units [2].

This solution presents an economic interest to the BR and to the aggregator as well (by selling this service).

The paper contains four sections; the next section introduces the binomial lattice approach before constructing the real option model that evaluates the load shedding services. The third section presents a direct application on a city scale case study; results and analysis are presented in the same section before concluding and proposing suggestions to improve the model in the last section.
REAL OPTION MODEL

The binomial lattice real option model approach was first proposed by Cox, Ross and Rubinstein in 1979 [4]. It is a discrete-time model that traces the evolution of the uncertainty in a discrete-time frame work. It is named binomial lattice, because at each time frame two possible futures are expected, each node represents a possible future with associated probability.

For any kind of binomial pricing approach there are three main steps to be done:

1- Model the uncertain future, which consists of building a tree of all possible futures with the associated probabilities. The random walk lattice equations are used in this work [4].

2- Value the real option (the right of using the load shedding switch) or the contract at exercising date. In this case at any possible time as long as the contract has not expired.

3- Discount the payoffs back to day 0, in order to determine the option price CP when the contract initiates.

In this work Tao Wang model is used; this model is a binomial lattice real option based model [5]. However, in addition to the presented three steps, Tao Wang evaluation process uses a decision optimization step [6]. This optimization step is further developed in this section.

Model Framework

As presented in the first section the aggregator is selling the load shedding services to the BR. The model seeks to evaluate the value CP of this service. By paying CP the BR obtains the right but not the obligation to shed some loads. It is assumed that the duration (h) and the access (a) to load shedding services are limited.

From the way this service is presented and from a financial perspective, it seems quite similar to an American call option contract [7] [8]; by paying a prime the buyer get the right but not the obligation to buy an agreed quantity of shares at a certain time for a certain price [7].

Finding the value of CP is the final objective of this study; the question this paper addresses can be rephrased as follows: under optimized decisions what is the maximum value the BR should be willing to pay for this contract?

Some assumptions are made in order to simplify the modeling process. The next sub-section lists them.

General Assumptions

1) The electrical heaters occupy a consumption ratio of $\alpha_2$ of the total consumption at any given time.

2) The network is fed from only two separated generation units $GU_1$ and $GU_2$.

3) The load profile is considered the only uncertain variable (it is modeled and explained later, and represented by the peak hour of each day).

4) The load shedding is considered like on/off switch that controls all the electrical heaters at once (all or nothing).

5) The controllable power all the time is $\alpha_2 \alpha_2$ of the total power consumption on the network as shown in Figure 2.

6) The duration of load shedding is one hour with no interruption and considered to be accepted by the end users.

7) The rebound effect that appears on the network once the load shedding switch is turned off is not considered in this first simplified model [9].

![Figure 2: Modified profile (just daily peaks)](image)

**General model**

**First step:** modeling the uncertain future.

Starting with a known value of $ED$ at day 0, two possible futures are expected either up $u.ED$ with probability of $p$ or down $d.ED$ with probability of $q$, ($u>1>d$), $u$, $d$, $p$ and $q$ are calculated using the equations of binomial lattice presented below:

$$u = e^{\sigma \sqrt{\Delta t}}$$
$$d = u^{-1}$$
$$p = \frac{e^{r \Delta t} - d}{u - d}$$
$$q = 1 - p$$

In this set of equations: $\Delta t$ is the time between the two followed time frame, $r$ is the discount rate and $\sigma$ is the volatility of the future electrical demand.

Figure 3 presents the first three time frames of the ED tree.

![Figure 3: Three steps ED tree](image)
At each node the shown value is the maximum value that may appear on the network and is considered to remain fixed for an entire hour. After the construction of the electrical demand tree, the corresponding modified electrical demand tree is constructed. The modification is made by turning the load shedding switch to on.

**Second step**: valuing the activation and the holding tree.
In order to optimize the use of the load shedding switch, especially because it is limited in number of use, a rational decision rules are considered and stated like follow:

- If $ED_i < \beta$ (fixed threshold) the BR will never use the load shedding switch; instead it will conserve it for hopefully more profitable future. Nevertheless this rule is not applicable the last day because there is no load shedding service afterwards.
- At any point of the timeline, the BR will have to choose between turning the switch of load shedding on (exercising the option) and conserving it (holding the option) for more attractive future.
- The value of the option at any point of the time line is the maximum between holding and exercising the option [5] [6].

As a result, for each future time line; there is one optimal moment where the payoff of activating the load shedding switch is maximal (Tao Wang optimization step). It is important to underline that this is not necessarily the highest ED value, because all these value are discounted back to day 0. Figure 4 shows ED future tree; the highlighted future is one possible future scenario; in this scenario the green node has the highest payoff. Which means in this possible scenario and to collect the maximum benefits from this contract the BR should activate its switch at the exact green node.

![ED tree, all possible future scenarios](image)

Same thing is applicable to all possible scenarios of the tree.

**Third step**: Valuing the option.
After creating the future, the exercising and holding trees, the last step consists of creating the real option value tree which represents the maximum value between the holding and the exercising tree. Lastly the values of the whole tree is the real option value at day 0 (load shedding service) and it is calculated as a sum of the nodes of the option value tree weighted by their probability and discounted back to day 0.

**CASE STUDY**
This numerical case study illustrates a possible contract between a national BR and a local aggregator with residential loads, at a city scale during the first 23 days of January.
The terms of the contract linking the two entities contains four major aspects.
Firstly: the contract life time is set to $n$ days during which the BR can use the load shedding switch for a limited time $a$. Secondly each time the load switch is activated, it sheds the all available controllable load for one hour without interruption. Thirdly the load shedding switch is accessible once a day maximum. Finally the contract price $CP$ has to be paid the first day.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$P_u$</th>
<th>$P_m$</th>
<th>GP1</th>
<th>GP2</th>
<th>$ED_0$</th>
<th>$\alpha_1$</th>
<th>$P_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>175</td>
<td>193</td>
<td>175</td>
<td>100</td>
<td>175</td>
<td>20%</td>
<td>100</td>
</tr>
<tr>
<td>Parameter</td>
<td>$\sigma$</td>
<td>$r$</td>
<td>$a$</td>
<td>$n$</td>
<td>$\Delta t$</td>
<td>$\alpha_2$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>Value</td>
<td>0.5</td>
<td>0.07</td>
<td>1</td>
<td>23d</td>
<td>24h</td>
<td>20%</td>
<td>200</td>
</tr>
</tbody>
</table>

**Table 1**: Case study values

Table 1 lists the numerical values for this case study. $ED_0$ represents the average of the peak hour electrical demand at first day expressed in MW; correspondingly $ED_i$ is the peak electrical demand that appears for one hour the day. {GP1, P1} and {GP2, P2} are the generation capacity MW and the energy price €/MWh for the first and the second generation source respectively, these four parameters are shown in Figure 5 below.

![Price €/MWh](image)

**Figure 5**: Energy price as function of generated power

First step: modeling the uncertain future.
With one step a day and a volatility of 50% Figure 3 transforms to Figure 6.
Second step: valuing the activation and the holding tree. Figure 7 illustrates an example of valuing the exercising option at a given time frame. In this example an electrical demand of 180MW is anticipated. 175MW are generated from the primary generation unite, while the other 5MW are generated from the expensive generation unit. This means if load shedding is used at this exact hour the $\alpha_1, \alpha_2 = 4\%$ represents 7.2MW. This controllable load is worth: $5 \times 200 + 2.2 \times 100 = 1220€$.

Third step: Valuing the option. The value of the option at any point of the timeline is the maximum value between using the load shedding switch or holding it for more profitable future, always respecting the established decision rules; in other words at any time it is the maximum between holding and exercising the option. Thus the option value tree is built through the exercising tree and the holding tree.

Once the option value tree is built, finally the price of the contract is the value of the whole tree discounted to first day. Figure 8 shows this final step on a three time intervals tree. The hunted value is $OV(1,1)$ which represents the value of the contract at days 0; it is calculated using the following equation from the right end side of the tree to left end side until reaching $OV(1,1):$ $OV(k,j) = (0.499 \times OV(k,j+1) + 0.501 \times OV(k+1,j+1)) \times e^{-r_1\Delta_1}$

As explained before $r$ is the discount rate and is assumed to be 0.07 while $\Delta_1$ is time between 2 steps of the tree here is taken one day. This means we look just at the maximum power that may appear on the network each day for one hour and then the BR takes the decision to activate or not the load shedding.

Results Analysis

According to the taken assumptions and the decision rules the service offered by the aggregator worth 752€ maximum for a one time switch during the 23 days of the contract. This means in a fair game the aggregator should not ask for more than 752€, in the same way the BR should never be willing to pay more than this amount. That is hedging risks coming from the electrical demand worth 752€. If somehow the aggregator asks more than that, taking risks is preferable form the BR point of view.

The following analysis explains the origins of the used evaluation technique.

As a matter of fact there are three dimensions for each one of these values $OV(i,j)$; the value itself, the probability of occurrence and the exact time the value occurs.

These three dimensions are essential in the modeling framework and they impact the final value of the contract.

The controllable loads power is 4% of the total electrical demand on the network. This means the controllable load power is bigger for higher electrical demand.

The ED, is the variable that walks randomly day after another, by one step a day either up with probability of 0.49 or down with probability of 0.51.

In other words a probability distribution curve can be constructed from the ED tree of Figure 6.

The ED probability distribution for the 23 days of the contract life time is presented is Figure 9. Because the final value is a discounted value of the weighted values of the tree, the time of occurrence of
each value is important; Figure 9 misses this information.

However, with less precision the final result can still be calculated by neglecting this information. From Figure 9 the probability of having values included in specific intervals can be calculated. This is done on Figure 10. In this figure four zones are presented:

- Zone1: The values of this zone are smaller than the fixed threshold 174MW; according to the decision rule1 if the EDi does not exceed 174MW the load shedding will not be activated, consequently in this zone the value of the load shedding contract is zero.
- Zone2: contains values between 175 and 174. If the load shedding is activated, the saved power should have been generated from the cheap generation unit.
- Zone4: its values are bigger than 182.3 = 175/ (1-0.04). In this zone if the load shedding is activated, the saved power should have been generated from the expensive generation unit.
- Zone3 contains values between 175 and 182.3 MW. If the load shedding is activate, the saved power should have been generated partially from the expensive unit and part from the cheap unit same as shown in Figure 5.

Figure 10 summarizes the 4 zones with the probability of each one of them; by applying the pricing formula as done in Figure 5 the option valuing function is established and shown in blue.

\[ OV = \sum (OV \ (\text{zone } i)) \ (\text{probability (zone } i)) = 780€. \]

This comes coherent with the value returned by the precise binomial lattice model 752 €. Of course such valuation is less precise due to the fact that:

1) Information about time of occurrence is not included
2) Probabilities per zone are average values, and this is not a lossless process.

CONCLUSION

This article has proposed a model to evaluate the load shedding services via real option theory; it has explained the general context of the problem and detailed the assumptions made to ensure better understanding of the problem. A general model has been developed based on the Tao Wang model which is a binomial lattice based approach and a case study has been presented as a direct numerical application of the developed model. The results of this study have been presented and analyzed.

In this article the only considered variable as uncertain is the electrical demand, which is a very simplified case of the real world; nevertheless it is a direct application for real option theory for smart grid applications that offers a promising and improvable solution.

Further work consists of improving the proposed model by eliminating some of the made assumptions to approach more from the real case with better precision, such as taking into account the rebound effect of the load shedding and multiple access per day; these assumptions may add complications to the model and may modify the final results.

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