MODELS OF *CVR* EVALUATION FOR DISTRIBUTION NETWORKS

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ABSTRACT

Reducing the supply voltage by a small percentage at the substation transformer (i.e. CVR) such that the utility and customer equipment may operate efficiently, while saving energy is now an accepted and proven concept. But it is still difficulty to exactly quantify the real energy saving in the distribution networks.
This paper aims to develop global and individual CVRs assessment models for distribution substations and loads.

INTRODUCTION

During the 80s, the electric utilities began studying the effect of the voltage reduction on energy usage on distribution networks. Indeed in 1981, EPRI gave a mandate to the University of Texas at Arlington for testing and analyzing the effects of reduced voltage on the efficiency of important power system loads (such as television sets, microwave ovens, motors, heat pumps, air conditioners, distribution transformers, resistance heating devices, etc.). In 1982, the Environmental Defence Fund (EDF) published an article discussing the efficacy of CVR as an energy conservation method.
From 1985 to 2000, electric utilities or organizations such as TEPCO (Tokyo Electric Power Company), Commonwealth Edison, NWPCC (Northwest Power Conservation Council, BPA (Bonneville Power Administration), Snohomosh County PUD, BC Hydro, PCS UtiliData, ESBN, etc., worked actively on the theory of the voltage reduction and the implementation of CVR.
From 2000, with the energy crisis in California, other companies and organizations were interested in energy efficiency by reducing the voltage distribution [1, 2].
The first HQ studies on the voltage lowering began in 2000, with the evaluation of the CVR at PBR distribution substation, in order to install in 2005 a voltage control prototype for energy saving. Before deploying this prototype to the key distribution substations, it was asked a confirmatory study on CVR values. Thus began, in 2011, a series of measurements on the voltage lowering at the substations Pte1, Pte2, Pte3 and Pte4.
The purpose of this paper is to develop models for the evaluation of distribution substation CVR, using electrical and meteorological quantities obtained from the measurements. The paper will first expose the theory of polynomial approximations for a function of *n* variables, using relative uncertainties to later develop models for the evaluation of global and individual CVRs, respectively for distribution substations and loads.

LEAST SQUARES APPROXIMATIONS

Polynomial approximation with one variable

Consider experimental points, resulting of the physical measurements (e.g. current versus voltage of a component). One might think to represent a function $y = f(x)$ by a straight line or a parabola (or other curve) by all points of physical measurements obtained. Denote by (x_i, y_i) the coordinates of points of experimental measurements, and choose $g(x)$ a function meant to represent, as closely as possible, the function $y = f(x)$ represented by the different experimental points of measures.
If we take $g(x)$ as a polynomial function of degree r, we get [3]

$$g(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_k x^k + \cdots + a_r x^r \qquad (1)$$

Denote by S the square of the difference between the experimental curve and the approximation curve, then it comes equation (2).

$$S = \sum_{i=0}^{n} \left[y_i - \left(a_0 + a_1 x + a_2 x^2 + \cdots + a_k x^k + \cdots + a_r x^r \right) \right]^2 \quad (2))$$

We will look for the coefficients $(a_0, a_1, a_2, \cdots, a_k, \cdots, a_r)$ so as to minimize S ; we can write $\partial S / \partial a_k = 0$. We get a matrix system of $(r+1)$ equations with $(r+1)$ unknowns presented below.

$$\begin{bmatrix} (n+1) & \sum x_1 & \sum x_i^2 & \cdots\cdots & \sum x_i^r \\ \sum x_1 & \sum x_i^2 & \sum x_i^3 & \cdots\cdots & \sum x_i^{r+1} \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \cdots\cdots & \sum x_i^{r+2} \\ & & \cdot & & \\ & & \cdot & & \\ & & \cdot & & \\ \sum x_i^r & \sum x_i^{r+1} & \sum x_i^{r+2} & \cdots\cdots & \sum x_i^{2r} \end{bmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ a_r \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ \cdot \\ \cdot \\ \cdot \\ \sum x_i^r y_i \end{pmatrix} \qquad (3)$$

with : $\sum = \sum_{i=1}^{n}$

Approximation by a function with *n* variables

Consider now the case of a function of several variables equal to four : this is the case of the voltage lowering where variation of the power P (or energy E) is a function of variations of the voltage V, those of the temperature T and those of the humidity H. And denote by

(x_i, y_i, z_i, w_i) the coordinate of the experimental points which w_i is the real/exact function; and by $g(x, y, z)$ the smoothing function.
If we look for the solution of the problem with respect to absolute uncertainties Δw_i, we obtain respectively the function S and the following matrix system [3, 4]:

$$S = \sum_{i=0}^{n} \left[w_i - \left(a_0 + a_1 x + a_2 y + a_3 z \right) \right]^2 \quad (4)$$

$$\begin{bmatrix} (n+1) & \sum x_i & \sum y_i & \sum z_i \\ \sum x_i & \sum x_i^2 & \sum x_i y_i & \sum x_i z_i \\ \sum y_i & \sum x_i y_i & \sum y_i^2 & \sum y_i z_i \\ \sum z_i & \sum x_i z_i & \sum y_i z_i & \sum z_i^2 \end{bmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_r \end{pmatrix} = \begin{pmatrix} \sum w_i \\ \sum x_i w_i \\ \sum y_i w_i \\ \sum z_i w_i \end{pmatrix} \quad (5)$$

Least squares approximation in relative values

Values are found scattered in general, as experimental results, subjected to uncertainties. When the absolute uncertainties Δw_i are almost constant throughout the range of variations of (x_i, y_i, z_i, w_i), the use of the above method is justified.

However, it happens that the relative uncertainty $\Delta w_i / w_i$ is constant, and the amplitude of variation of w_i is great in the range of variations of (x_i, y_i, z_i, w_i). Then, the use of method (5) leads to favour the values of w_i with high modules with respect to those with low modules. It is then necessary to minimize *S*, not in relation to the absolute differences, but from the relative differences $\Delta w_i / w_i$
Then it comes the equation of *S* and the matrix solution [3]

$$S = \sum_{i=0}^{n} \left[\frac{w_i - \left(a_0 + a_1 x + a_2 y + a_3 z \right)}{w_i} \right]^2 \quad (6)$$

$$\begin{bmatrix} \frac{(n+1)}{w_i^2} & \sum\left(\frac{x_i}{w_i^2}\right) & \sum\left(\frac{y_i}{w_i^2}\right) & \sum\left(\frac{z_i}{w_i^2}\right) \\ \sum\left(\frac{x_i}{w_i^2}\right) & \sum\left(\frac{x_i^2}{w_i^2}\right) & \sum\left(\frac{x_i y_i}{w_i^2}\right) & \sum\left(\frac{x_i z_i}{w_i^2}\right) \\ \sum\left(\frac{y_i}{w_i^2}\right) & \sum\left(\frac{x_i y_i}{w_i^2}\right) & \sum\left(\frac{y_i^2}{w_i^2}\right) & \sum\left(\frac{y_i z_i}{w_i^2}\right) \\ \sum\left(\frac{z_i}{w_i^2}\right) & \sum\left(\frac{x_i z_i}{w_i^2}\right) & \sum\left(\frac{y_i z_i}{w_i^2}\right) & \sum\left(\frac{z_i^2}{w_i^2}\right) \end{bmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_r \end{pmatrix} = \begin{pmatrix} \sum\left(\frac{1}{w_i}\right) \\ \sum\left(\frac{x_i}{w_i}\right) \\ \sum\left(\frac{y_i}{w_i}\right) \\ \sum\left(\frac{z_i}{w_i}\right) \end{pmatrix} \quad (7)$$

The matrix systems (5) and (7) represent respectively the mathematical model of CVR using the absolute and the relative uncertainties. In this study, we will use the second method.

MODELS OF *CVR* EVALUATION

We will first propose the mathematical model for evaluating the global CVR of the distribution substation and then develop the individual CVR of the classes of load.

Global CVR assessment model of the substation

Before deploying the voltage control prototype to the key distribution substations of HQD, we realized an evaluation of CVR confirmatory study of some fundamental substations. Thus began in 2011, a campaign of measurements on the voltage lowering in several substations among which Pte1, Pte2, Pte3, and Pte4.

During this voltage lowering campaign, the chosen approach consists in inserting between not-lowering days, a day of voltage reduction, during the period of the days of the week and the period of the days of weekend or holidays. Also, during each day of lowering or not-lowering, are measured electrical quantities (current I, voltage V, active/reactive power P/Q) and meteorological parameters (temperature T, and humidity H).

After obtaining the data measurements and their treatment, we pass to the evaluation of the active/reactive (CVR_p / CVR_q) global CVRs of each substation and then determine individual CVRs (i.e. the individual coefficients n_p and n_q) of load classes. For this, are first calculated, for each measured quantity (or parameter) Y_i, the following amounts $\Delta Y_i = Y_{i+1} - Y_i$ and $\tilde{\Delta} Y_i = \Delta Y_i / Y_i$ that will be used in equation (7) to obtain the matrix system (8) for active *P* and reactive *Q* powers.

$$\begin{bmatrix} \sum_{i=1}^{n}\left(\tilde{\Delta}V_i\right)^2 & \sum_{i=1}^{n}\left(\tilde{\Delta}V_i \cdot \tilde{\Delta}T_i\right) & \sum_{i=1}^{n}\left(\tilde{\Delta}V_i \cdot \tilde{\Delta}H_i\right) \\ \sum_{i=1}^{n}\left(\tilde{\Delta}V_i \cdot \tilde{\Delta}T_i\right) & \sum_{i=1}^{n}\left(\tilde{\Delta}T_i\right)^2 & \sum_{i=1}^{n}\left(\tilde{\Delta}T_i \cdot \tilde{\Delta}H_i\right) \\ \sum_{i=1}^{n}\left(\tilde{\Delta}V_i \cdot \tilde{\Delta}H_i\right) & \sum_{i=1}^{n}\left(\tilde{\Delta}T_i \cdot \tilde{\Delta}H_i\right) & \sum_{i=1}^{n}\left(\tilde{\Delta}H_i\right)^2 \end{bmatrix} \bullet \begin{bmatrix} a_{p1} \\ a_{p2} \\ a_{p3} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}\left(\tilde{\Delta}V_i \cdot \tilde{\Delta}P_i\right) \\ \sum_{i=1}^{n}\left(\tilde{\Delta}T_i \cdot \tilde{\Delta}P_i\right) \\ \sum_{i=1}^{n}\left(\tilde{\Delta}H_i \cdot \tilde{\Delta}P_i\right) \end{bmatrix}$$

(8a)

$$\begin{bmatrix} \sum_{i=1}^{n}\left(\tilde{\Delta}V_i\right)^2 & \sum_{i=1}^{n}\left(\tilde{\Delta}V_i \cdot \tilde{\Delta}T_i\right) & \sum_{i=1}^{n}\left(\tilde{\Delta}V_i \cdot \tilde{\Delta}H_i\right) \\ \sum_{i=1}^{n}\left(\tilde{\Delta}V_i \cdot \tilde{\Delta}T_i\right) & \sum_{i=1}^{n}\left(\tilde{\Delta}T_i\right)^2 & \sum_{i=1}^{n}\left(\tilde{\Delta}T_i \cdot \tilde{\Delta}H_i\right) \\ \sum_{i=1}^{n}\left(\tilde{\Delta}V_i \cdot \tilde{\Delta}H_i\right) & \sum_{i=1}^{n}\left(\tilde{\Delta}T_i \cdot \tilde{\Delta}H_i\right) & \sum_{i=1}^{n}\left(\tilde{\Delta}H_i\right)^2 \end{bmatrix} \bullet \begin{bmatrix} a_{q1} \\ a_{q2} \\ a_{q3} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}\left(\tilde{\Delta}V_i \cdot \tilde{\Delta}Q_i\right) \\ \sum_{i=1}^{n}\left(\tilde{\Delta}T_i \cdot \tilde{\Delta}Q_i\right) \\ \sum_{i=1}^{n}\left(\tilde{\Delta}H_i \cdot \tilde{\Delta}Q_i\right) \end{bmatrix}$$

(8b)

with : $\tilde{\Delta} P_i = \Delta P_i / P_i$, $\tilde{\Delta} Q_i = \Delta Q_i / Q_i$, $\tilde{\Delta} V_i = \Delta V_i / V_i$, $\tilde{\Delta} T_i = \Delta T_i / T_i$ and $\tilde{\Delta} H_i = \Delta H_i / H_i$

$[a_{p1} \; a_{p2} \; a_{p3}]^t$ - vector of active *CVRp*

$[a_{q1} \; a_{q2} \; a_{q3}]^t$ - vector of reactive *CVRq*.

Individual CVR assessment model of the load

Consider the case of a distribution substation with its feeders feeding loads formed by residential, commercial and industrial.
Consider the model of *static exponential* of the load taking into account the knowledge of the composition of the load in its various classes mentioned above.
One can write the following equation to define a distribution feeder/substation [5].

$$P_k = P_{R0}\left(\frac{V_0-\Delta V}{V_0}\right)^{n_{p(R)}} + P_{NR0}\left(\frac{V_0-\Delta V}{V_0}\right)^{n_{p(NR)}} + \cdots$$
$$\cdots + P_{C0}\left(\frac{V_0-\Delta V}{V_0}\right)^{n_{p(C)}} + P_{I0}\left(\frac{V_0-\Delta V}{V_0}\right)^{n_{p(I)}}$$

$$P_k = \frac{R\%}{100}\cdot P_{ko}\left(\frac{V_0-\Delta V}{V_0}\right)^{n_{p(R)}} + \frac{NR\%}{100}\cdot P_{ko}\left(\frac{V_0-\Delta V}{V_0}\right)^{n_{p(NR)}} + \cdots$$
$$\cdots + \frac{C\%}{100}\cdot P_{ko}\left(\frac{V_0-\Delta V}{V_0}\right)^{n_{p(C)}} + \frac{I\%}{100}\cdot P_{ko}\left(\frac{V_0-\Delta V}{V_0}\right)^{n_{p(I)}}$$

Then, we find equation (9).

$$P_k = \alpha_R\cdot P_{k0}\left(1-\frac{\Delta V}{V_0}\right)^{n_{p(R)}} + \alpha_{NR}\cdot P_{k0}\left(1-\frac{\Delta V}{V_0}\right)^{n_{p(NR)}} + \cdots$$
$$\cdots + \alpha_C\cdot P_{k0}\left(1-\frac{\Delta V}{V_0}\right)^{n_{p(C)}} + \alpha_I\cdot P_{k0}\left(1-\frac{\Delta V}{V_0}\right)^{n_{p(I)}} \quad (9)$$

with :

$\alpha_R = (R\%)/100$; $\alpha_{NR} = (NR\%)/100$;
$\alpha_C = (C\%)/100$; $\alpha_I = (I\%)/100$;
and $(\alpha_R + \alpha_{NR} + \alpha_C + \alpha_I) = 1$

If we make the following approximation [4] :

$$\left(1-\frac{\Delta V}{V_0}\right)^{n_p} \simeq 1 - n_p\cdot\frac{\Delta V}{V_0}$$

it comes then :

$$\frac{(\Delta P_k/P_{k0})}{(\Delta V/V_0)} = \left(\alpha_R\cdot n_{p(R)} + \alpha_{NR}\cdot n_{p(NR)} + \alpha_C\cdot n_{p(C)} + \alpha_I\cdot n_{p(I)}\right)$$

$$CVR_k = \left(\alpha_R\cdot n_{p(R)} + \alpha_{NR}\cdot n_{p(NR)} + \alpha_C\cdot n_{p(C)} + \alpha_I\cdot n_{p(I)}\right) \quad (10)$$

Equation (10) expresses the active global CVR_p of a distribution substation depending on the load composition and on the coefficients n_p representing the individual CVR_p of each class of the overall load of the substation (or feeder).

To determine the unknowns $\left(n_{p(R)}, n_{p(NR)}, n_{p(C)}, n_{p(I)}\right)$ that are the individual CVR_p of each class of loads, knowing the global CVR_p of the substation/feeder, one should find four substations lying in the same weather conditions (i.e. temperature, humidity and wind) and having identical classes of load, in order to establish a system of *n* equations with *n* unknowns. Finally, it provides the matrix system (11) linking the global CVR_p of substations with the coefficients n_p of the individual classes of loads.

$$\begin{bmatrix} \alpha_{R1} & \alpha_{NR1} & \alpha_{C1} & \alpha_{I1} \\ \alpha_{R2} & \alpha_{NR2} & \alpha_{C2} & \alpha_{I2} \\ \alpha_{R3} & \alpha_{NR3} & \alpha_{C3} & \alpha_{I3} \\ \alpha_{R4} & \alpha_{NR4} & \alpha_{C4} & \alpha_{I4} \end{bmatrix} \cdot \begin{pmatrix} n_{p(R)} \\ n_{p(NP)} \\ n_{p(C)} \\ n_{p(I)} \end{pmatrix} = \begin{pmatrix} CVR_{k1} \\ CVR_{k2} \\ CVR_{k3} \\ CVR_{k4} \end{pmatrix} \quad (11)$$

The matrix equation (11), of the form [A]*(x) = (b), is the model of the individual *CVRp* (or coefficients n_p) of the classes of loads.

EXPERIMANTAL CURVES AND RESULTS OF SIMULATIONS

Results of measurements

We present in figures 1 to 4, some curves obtained during the winter campaign of measurements.

The green and blue curves represent the measured voltages (during 24 hours), respectively during a day of not lowering and a day of lowering, for the substation Pte1 in the case of figure 1; and for the substation Pte3 in the case of figure 3.
The green and blue curves are the measured active powers (during 24 hours), respectively during a day of not lowering and a day of lowering, for the substation Pte1 in the case of figure 2; and for the substation Pte3 in the case of figure 4.

Results of simulation

Table 1 shows the composition of the load (residential, commercial and industrial) for the four distribution substations where the measurements were made.
We present in Table 2 the computed values (from matrix system 8a) of the four distribution substations *CVRp*, for the winter time.
From matrix system (11), we obtain the computed *CVRp* values (also called the n_p coefficients) of the residential, commercial and industrial loads; for winter time. These values are shown in Table 3.

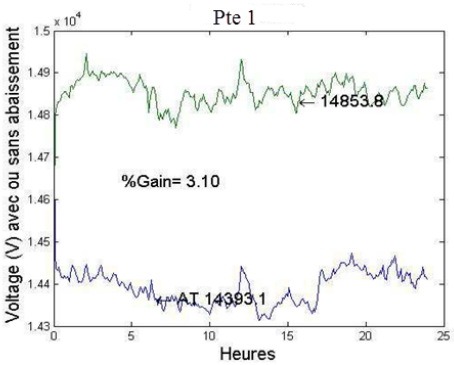

Figure 1 Voltage curves during voltage not-lowering and lowering

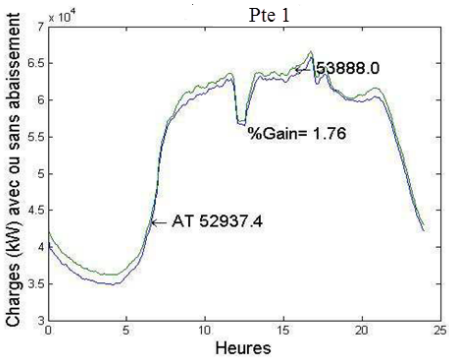

Figure 2: Curve of P during voltage not-lowering and lowering

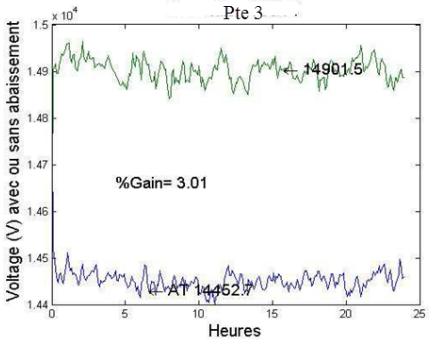

Figure 3: Voltage curves during voltage not-lowering and lowering

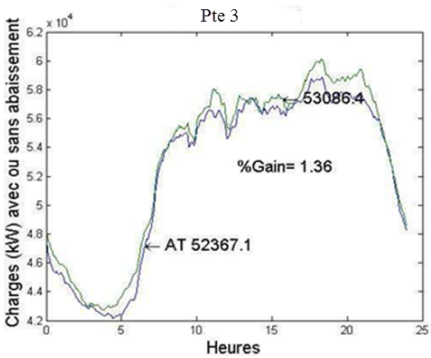

Figure 4: Curve of P during voltage not-lowering and lowering

Table 1: Data of the load composition

Substation	Residential	Commercial	Industrial
Pte1 [T1 & T2]	60.6%	31.9%	7.5%
Pte2 : [T3]	33%	62.7%	4.2%
Pte3 [T3 & T4]	39.1%	19.4%	41.5%
Pte4 : [T1 & T2]	64.5%	30.3%	5.2%

Table 2: Values of global *CVRp* for the 4 substations

Substation	Connection Scheme	Computed *CVRp*
Pte1	T1&T2	0.6653
Pte2	T3	0.6844
Pte3	T3&T4	0.4814
Pte4	T1&T2	0.6449

Table 3: Values of individual *CVRp* for the 3 load classes

		Individual CVR(n_p)
Load classes	Residential (TAE+NTAE)	0.6610
	Commercial	0.7349
	Industrial	0.1961

FUTURE WORK

Future research is devoted to deepening the models of global active *CVRp* and to developing the models of global reactive *CVRq*, before focusing on the evaluation of the *dynamic* CVR of a substation that is directly connected to a wind farm.

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