A PROBABILISTIC APPROACH FOR OPTIMAL CAPACITOR PLACEMENT IN A DISTRIBUTION SYSTEM USING SIMULTANEOUS PERTURBATION STOCHASTIC APPROXIMATION

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ABSTRACT

This paper deals with the problem of choosing optimal locations and sizes of capacitors in three-phase unbalanced distribution systems. This is a mixed, non-linear, constrained optimization problem that must be formulated in probabilistic scenarios to take into account the unavoidable uncertainties that affect the problem's input data. To reduce the computational efforts, a new optimization model with only inequality constraints is formulated and the Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm is applied. The proposed approach is tested on the IEEE 34-node unbalanced distribution systems.

INTRODUCTION

Shunt capacitors are used in distribution systems for reducing the power losses and improving the voltage profile along the feeders. This paper investigates the problem of their optimal locations and sizes in unbalanced distribution systems.
This problem is usually formulated as a mixed integer, non-linear, constrained optimization problem with both equality and inequality constraints and solved in deterministic scenarios by applying heuristic techniques, such as the Genetic Algorithms (GA), as a solution method [1]. However, the time-varying nature of loads leads to inaccurate deterministic solutions and, then, a probabilistic optimization model is more adequate [2].
The classical Monte Carlo simulation procedure in the frame of a GA requires tremendous and unacceptable computational efforts. Then, to reduce the computational burden, in [2] two different techniques based on the linearized form of the equality and inequality constraints of the probabilistic optimization model and on the Point Estimate Method were proposed.
In this paper, to further reduce computational efforts, a highly performing algorithm based on the Simultaneous Perturbation Stochastic Approximation (SPSA) [3] is proposed to solve the optimization problem. SPSA is widely used to solve optimization problems in several areas.
SPSA can be easily implemented and allows an efficient gradient approximation. It operates perturbing all problem unknowns simultaneously (both continuous and integer) and approximating differentiation stochastically during iterations. Although SPSA algorithm is a stochastic optimization solution method, simultaneous perturbation and population-free make the computation very effective.
SPSA algorithm solves successfully and with reduced computational efforts optimization problems. In this paper, the problem of the optimal allocation in unbalanced distribution systems is transformed into a mixed integer, non-linear, optimization problem with inequality constraints.
Eventually, this paper is organized as follows. Firstly the mathematical formulation of the probabilistic optimization problem of the sizing and siting of capacitors with both equality and inequality constraints is briefly recalled. Then, the transformed problem including only inequality constraints is formulated. Finally, the SPSA algorithm is described and tests on an IEEE unbalanced test feeder are presented and discussed.

OPTIMIZATION MODEL WITH EQUALITY AND INEQUALITY CONSTRAINTS

When the problem of the location and sizes of capacitors is dealt with, a mixed integer, non-linear, constrained optimization model is formulated which takes a number of deterministic load levels as references.
Recently, in the relevant literature [2], it was shown that the time-varying nature of the load demands inside each time interval, makes mandatory a probabilistic formulation that considers the input variables (mainly phase-load demands) to be random variables. The load powers random variations versus time are characterized by normal distributions and the problem of the optimal allocation of capacitors is formulated in term of probabilistic objective function f_{obj} and equality g_m, inequality h_j constraints as [2]:

$$\min f_{obj}(\mathbf{B},\mathbf{C}) \tag{1}$$

subject to:

$$g_m(\mathbf{B},\mathbf{C}) = 0\,, \quad m = 1,\ldots\ldots N_{ec} \tag{2}$$

$$h_j(\mathbf{B},\mathbf{C}) \le 0\,, \quad j = 1,\ldots\ldots N_{ic} \tag{3}$$

where **B** is the system state vector (magnitudes and arguments of the voltages, at all the busbars) and **C** is the control vector, related to fixed/switched capacitors to be placed at each bus; N_{ec} and N_{ic} are the number of equality and inequality constraints, respectively.
The *objective function* in (1) is the total expected costs

(capacitors and losses) defined as:

$$f_{obj} = Cost_C + \mu\left[Cost_L\right] \qquad (4)$$

where: $\mu\,[Cost_L]$ is the expected value of the losses cost; $Cost_C$ is the capacitor cost.
The *equality constraints* to be satisfied are the three-phase probabilistic load-flow equations that can be expressed synthetically as:

$$\mathbf{g_1}\,(\mathbf{Y}) = \mathbf{U}, \qquad (5)$$

where: **U** is the input random vector (active and reactive load phase powers and active three-phase generation powers) and **Y** is the state random vector (magnitude and argument of the unknown phase-voltages).
Also the equations linking dependent to state variables have to be included in the equality constraints:

$$\mathbf{D} = \mathbf{g_2}\,(\mathbf{Y}), \qquad (6)$$

where **D** is the dependent variables random vector. In this paper, the dependent variables are the power losses, the line currents and the unbalance factors.
Details of the explicit form of the power flow equations and the dependent variables equations are reported in well known power system analysis textbooks.
The *inequality constraints* are linked to the line current maximum values that should not exceed the line ratings, and the unbalance factors 95th percentiles that should not exceed the allowable value provided by the power quality standards [4]. Moreover, under normal operating conditions, during each period of one week, 95% of the mean rms values of the supply voltage shall be within the range of ±10% of the declared voltage [4].
Then, the inequality constraints include [2]:

$$\begin{aligned}
&\mu(I_l) + 3\sigma(I_l) \le I_{l,\max} && l \in \Omega_l \\
&\int_0^{k_{d,\max}} f_{k_{d,i}}\, dk_{d,i} \ge 0.95 && i \in \Omega_{3p} \\
&\int_{V_{\min}}^{V_{\max}} f_{V_{i,p}}\, dV_{i,p} \ge 0.95 && i = 1,\ldots,N_{bus}\,,\; p = 1,\ldots,N_{ph,i}
\end{aligned} \qquad (7)$$

where: $\mu(I_l)$ and $\sigma(I_l)$ are the expected values and the standard deviation of the current at line *l*, respectively; $I_{l,\max}$ is the current rating for line *l*; Ω_l is the set of system lines; $k_{d,i}$ is the unbalance factor at bus *i*; $f_{k_{d,i}}$ is the probability density function (pdf) of $k_{d,i}$; $k_{d,\max}$ is the maximum unbalance factor; Ω_{3p} is the set of three-phase busbars; $f_{V_{i,p}}$ is the pdf of $V_{i,p}$ (amplitude of voltage at phase *p* of busbar *i*), and V_{min}, V_{max} indicate the admissible range of voltages; $N_{ph,i}$ is the number of phases at busbar *i*, and N_{bus} is the number of busbars.

OPTIMIZATION MODEL WITH ONLY INEQUALITY CONSTRAINTS

SPSA algorithm solves successfully and with reduced computational efforts optimization problems characterized by *only inequality constraints*. Then, the optimization problem (1) – (3) must be transformed in order to include the equality constraints (2) in the objective functions and in the inequality constraints. The equality constraints to be satisfied are the three-phase probabilistic load-flow equations (5) and the equations linking dependent to state variables (6). These equations can be eliminated: (i) linearizing equality constraints (5) around an expected value region in order to obtain state variables versus input data in closed form[1]; (ii) including the results of step (i) in (6) to obtain dependent variables in closed form; and (iii) including the results of steps (i) and (ii) in the objective functions (4) and inequality constraints (7) to eliminate the equality constraints presence.
With reference the above step (i), let the vector **μ**(**U**) be the expected values of **U** in (5). If a deterministic three-phase load flow is calculated using **μ**(**U**) as input data, the solution of (5) in the field of the deterministic model will be given by the vector $\mathbf{Y}_0$, such that:

$$\mathbf{g_1}\,(\mathbf{Y}_0) = \boldsymbol{\mu}(\mathbf{U}). \qquad (8)$$

Linearizing non-linear system (5) around point $\mathbf{Y}_0$ and considering relationship (8), we obtain:

$$\mathbf{Y} \cong \mathbf{Y'}_0 + \mathbf{A}\,\mathbf{U}, \qquad (9)$$

where:

$$\mathbf{A} = \left[\left. \frac{\partial \mathbf{g}_1}{\partial \mathbf{Y}} \right|_{\mathbf{Y} = \mathbf{Y}_o} \right]^{-1}, \qquad \mathbf{Y'}_0 = \mathbf{Y}_0 - \mathbf{A}\,\boldsymbol{\mu}(\mathbf{U}).$$

Relationships (9) express each element of the state vector **Y** as a linear combination of the elements of the input vector **U**. It represents the needed relationships for the above step (i). Abovementioned steps (ii) and (iii) are trivial to be performed.

SOLUTION PROCEDURE

The probabilistic optimization problem formulated before to obtain the optimal siting and sizing of capacitors is a large-scale, mixed, non-linear constrained optimization problem. A solving procedure in two steps is proposed.

In order to reduce the processing time while maintaining reasonable accuracy, the search space can be reduced. This can be achieved by means of sensitivity structures that can determine a reduced feasible region when dealing with continuous variables or with a limited set of candidate solutions when dealing with discrete variables. Therefore, the optimization problem has two steps: the first step is to determine the reduced feasible

[1] Since the input variables are Gaussian- distributed, the input variables are also Gaussian- distributed, and, then, the pdfs' and the statistical parameters in (7) can be expressed in closed-form relationships.

region and the second is to find the optimal solution by the SPSA algorithm.

With reference to the first step, several techniques aimed at individuating the reduced feasible region can be exploited [1]. In this paper, the reduced feasible region is obtained by applying the Inherent Structure Theory of Networks (ISTN) that defines a simplified structure that is useful in describing the sensitivity of the state variables with system changes [5].

The proposed solving procedure is then based on the following steps:

1. application of ISTN to individuate a set of candidate busbars;
2. application of the SPSA algorithm to solve the optimization problem; the set of candidate solutions for siting is given by the set of candidate busbars determined in step 1, and the set of candidate solutions for sizing is given by the available standard ratings for capacitors.

Brief remarks on ISTN

In the frame of the ISTN [5], it has been demonstrated that the spectral representation of the impedance matrices - direct or inverse sequence impedance matrix at fundamental frequency or harmonic impedance matrices - defines the inherent system sensitivity structure which helps in describing the sensitivity to circuit variables. When dealing with capacitor placement in unbalanced systems, the sensitivity structure is derived with reference to the three-phase impedance matrix at fundamental frequency [1].

Let us consider an unbalanced power system and its three-phase admittance matrix that is a $N_{ph,tot}$ matrix. In [1] it has been proved that the 2-norm of the voltage vector $\mathbf{V}$ (that is a $N_{ph,tot}$ vector) can be evaluated by the following approximate expression:

$$\|\mathbf{V}\| \approx \left(\dot{\lambda}_k^s\right)^{-1} \left|\left(\mathbf{\Gamma}_k^s\right)^H \mathbf{I}\right| \tag{10}$$

being $\dot{\lambda}_k^s$ the eigenvalue of the three-phase admittance matrix that has the minimum modulus (corresponding to the bus k with phase s), $\mathbf{\Gamma}_k^s$ the corresponding reciprocal eigenvector, $\mathbf{I}$ the current vector.

The approximate relationships that were obtained can be useful in identifying the candidate busbars for the location of capacitors, having in mind that the placement of a capacitor should result in voltage variations. Identification can be achieved using the elements of the sensitivity matrix of the eigenvalue $\dot{\lambda}_k^s$,referred to as $\dot{\mathbf{S}}_k^s$. In particular, each element of this matrix represents the sensitivity of the eigenvalue $\dot{\lambda}_k^s$ with respect to the elements of the three-phase admittance matrix. Having in mind that the voltages at all busbars are linked to the the eigenvalue $\dot{\lambda}_k^s$, (see (10)) and that the placement of a capacitor at any system busbar causes a variation of only a self-admittance term of the admittance matrix, the set of candidate busbars can be selected as the ones that correspond to the highest values of the diagonal elements of the sensitivity matrix $\dot{\mathbf{S}}_k^s$.The selection of the candidate busbars can be performed, thus, by choosing the busbars with the highest values of $S_k^s(i,i)$.

Further details can be found in [1].

THE SPSA ALGORITHM

The Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm, in the integer version, solves the optimization problem:

$$\min f(\mathbf{X}) \tag{11}$$

subject to:

$$z_j(\mathbf{X}) \le 0, \qquad j = 1, \ldots\ldots, N_{ic} \tag{12}$$

with

$\mathbf{X} = [x_1 \ldots, x_q]^T$

including q integer variables.

This algorithm operates perturbing all unknowns simultaneously and approximating differentiation stochastically during iterations [3].

Let us consider initially the unconstrained problem and let us define $\lceil . \rceil$ to be the operator that rounds a real number to the next integer of larger magnitude.

SPSA algorithm searches the minimum of the objective function f in (11) updating the components of vector $\mathbf{X}$ at $(k+1)^{th}$ iteration as follows [6]:

$$\mathbf{X}_{k+1} = \mathbf{X}_k - \lceil a_k \mathbf{g}_k \rceil \mathbf{\Delta}_k \, , \tag{13}$$

being:

$$\mathbf{g}_k = \frac{f(\mathbf{X}_k + c_k \mathbf{\Delta}_k) - f(\mathbf{X}_k - c_k \mathbf{\Delta}_k)}{2c_k} \begin{bmatrix} (\Delta_{k1})^{-1} \\ \vdots \\ (\Delta_{kq})^{-1} \end{bmatrix} \tag{14}$$

In (13) and (14) k is the iteration index, the multiplier a_k is a constant for accelerating convergence which depends on the expected maximum number of iterations, the expected step size and the starting values of the optimization variables; $\mathbf{\Delta}_k$ is the q-dimensional random perturbation vector, whose components are independently generated following a Bernoulli (±1) distribution [6]; c_k is a decayed number. Detail about the values of these quantities are given in [6].

The extension to the inequality constrained case requires easy modifications of (13) to include into the updating of the vector $\mathbf{X}$ components the effect of violated constraints. Details are omitted here for the sake of brevity and fully reported in [7].

NUMERICAL APPLICATIONS

The problem of sizing and siting of capacitors has been solved for the unbalanced IEEE 34-bus test system shown in Fig. 1 [8] where the original capacitor banks and voltage regulators have been removed. This system contains a mixture of single- and three-phase lines and loads, making it also quite suitable for testing the SPSA algorithm. The complete network data and parameters can be found in [8].

The load demands are uncorrelated Gaussian-distributed random variables. The mean values of the load powers are assumed to be percentages of the peak level reported in [8] while the standard deviation was assumed to be 10%. We considered two case study, defined as:

- Case 1: the mean values of the load powers were assumed to be 75 % of the peak level;
- Case 2: the mean values of the load powers were assumed to be 100% of the peak level;

In reference to the constraints, the maximum line currents were fixed at the ratings reported in [8], and the maximum value of the 95th percentile of the unbalance factor was assumed to be 3%. With respect to the voltage at each busbar, only the maximum value of the voltage was constrained (i.e., the 95th percentile of the voltage was assumed to be less than 110% of the nominal value). The unit capacitors available at any bus were assumed to come in discrete sizes of 25 kVar to enlarge the search space.

The application of ISTN has given the following set of candidate busbars:

Ω ={838, 840, 862, 836, 848, 846, 860}.

The results were compared with the results obtained by applying :

- the probabilistic approach based on the use of a micro Genetic Algorithm (μGA) of [2];
- the Exhaustive Search: each possible solution (in terms of candidate busbars and installed reactive power of capacitors) is tested. In particular, a maximum value of reactive power (i.e., 200 kVAr) of installed capacitors for each busbar phase was imposed.

The simulations were carried out with Matlab programs by a 3.6 GHz PC Xeon processor E3-1280v2.

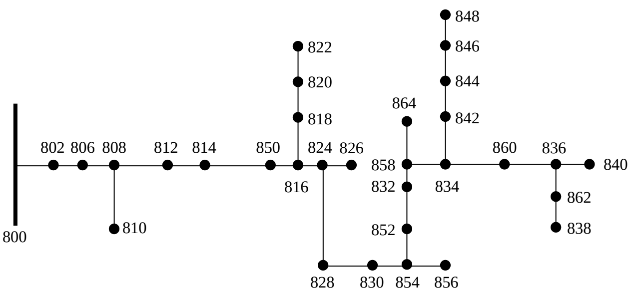

Fig. 1 - IEEE 34-bus distribution test system [8]

Tab. I shows the optimal placements and sizes of capacitors obtained with SPSA, micro-Genetic algorithms and Exhaustive Method in case 1. The objective function is normalized with respect to the value that it assumes when no capacitors are installed.

Table I: Optimal capacitor location and size in case 1

Method	Typology, location and size of capacitors	Objective function [pu]
SPSA	Three-phase capacitor banks: 75 kVar @ #848 75 kVar @ #860 150 kVar @ #836 150 kVar @ #840 150 kVar@ #862	0.7558
μGA	Three-phase capacitor banks: 300 kVar @ #846 450 kVar @ #860	0.7458
Exhaustive	Three-phase capacitor banks: 150 kVar @ #836 300 kVar @ #846 300 kVar @ #860	0.7458

Tab. II shows the capacitor optimal placements and sizes obtained with SPSA, micro Genetic algorithms and Exhaustive Method in case 2.

Table II: Optimal capacitor location and size in case 2

Method	Typology, location and size of capacitors s	Objective function [pu]
SPSA	Three-phase capacitor banks: 450 kVar @ #846 150 kVar @ #848 225 kVar @ #860 225 kVar @ #836 450 kVar @ #840 150 kVar @ #862 Single-phase capacitor banks: 50 kVar @ #838	0.7979
μGA	Three-phase capacitor banks: 600 kVar @ #846 600 kVar @ #860	0.7743
Exhaustive	Three-phase capacitor banks: 600 kVar @ #846 600 kVar @ #860	0.7743

Finally, Table III compares the computational efforts, assuming 1 p.u. the time required by the μGA based method.

Table III: Computational efforts

Method	Computational Time [p.u.]	
	case 1	case 2
SPSA	0.0096	0.0277
μGA	1	1
Exhaustive	28.9	20.41

From the analysis of Tables I, II and III the following considerations arise.

- in case 1, the method based on SPSA found a solution with a value of the objective function that is greater than the optimal value found by the μGA based- and the exhaustive methods. However, it is worth noting that the absolute value of the difference is 0.01 pu leading to a relative difference of 1.3 %.
- The solution with an objective function greater of 1.3 % was found in a computational time much smaller (0.0096 versus 1 for μGA, and versus 20.41 for the exhaustive method).
- The results of case 2 allow to make similar considerations.
- In case 1, the total reactive power determined by SPSA method is smaller than the one determined by other methods (600 kVar versus 750 kVar).
- In case 2, the total reactive power determined by SPSA method is higher than the one determined by other methods (1650 kVar plus 50 kVar of single-phase capacitors versus 1200 kVar).

If we stop the μGA at the computational time required by SPSA based method, we obtain the values of the objective function that are reported in Table IV. Different considerations arise in case 1 and in case 2. In fact, in case 1, if we stop the μGA at time 0.0096 pu (see Table III), the optimal value of the objective function is less than the optimal value found by SPSA (0.7476 pu versus 0.7558 pu).

In case 2, the μGA found a solution worse than the optimal solution found by SPSA (0.8261 pu versus 0.7979 pu).

A possible reason is that in case 2 the initial population of the μGA (randomly chosen) have generally bad fitness because of the high probability to have constraint violations; so the μGA cannot be fast as the case 1 were no violations was found in the initial population.

Table IV: Values of the objective function found by μGA at the SPSA computational time

Case	Objective function [pu]
1	0.7476
2	0.8261

CONCLUSIONS

In this paper, a highly performing algorithm was applied to solve the problem of the optimal sizing and siting of capacitors in unbalanced distribution networks. The proposed algorithm, i.e. the Simultaneous Perturbation Stochastic Approximation algorithm, handles contemporaneously both continuous and integer variables and approximates differentiation stochastically during iterations.

The optimal capacitor allocation model was transformed in a mixed integer, non-linear, constrained optimization problem with only inequality constraints as required by the proposed algorithm.

The results obtained applying SPSA have been compared with the results obtained with other methods previously implemented. The main outcomes of the paper are that the Simultaneous Perturbation Stochastic Approximation algorithm:

i. is robust in solving the capacitor optimal allocation problem and gives outputs that are comparable to those produced by other methods;
ii. is appropriate for applications that require fast time response, guaranteeing computational efforts much less than the efforts required by methods based on the application of Genetic Algorithms.

Further research is in progress to extend the application of SPSA algorithm to the case of the optimal allocation of capacitors and voltage regulators in a three-phase unbalanced distribution system.

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