

## FUZZY OPTIMAL PLACEMENT OF CAPACITORS IN THE PRESENCE OF NONLINEAR LOADS IN UNBALANCED DISTRIBUTION NETWORKS USING GSA ALGORITHM

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### ABSTRACT

*This paper presents a new method to optimize locating and the size of fixed and switching capacitor banks based on Gravitational Search Algorithm (GSA). The algorithm is proposed for radial and meshed networks in the presence of unbalanced and nonlinear loads. The objective function considers the minimization of the total energy loss costs and capacitor installation cost, network total harmonic distortion (THD) index, and the deviation of the voltage fundamental component from the permitted value. Since these parameters do not have similar units and variation ranges, a membership degree is assigned to each parameter by using the fuzzy sets. The simulation results tested on the IEEE 123-Bus distribution network confirm the efficiency of the proposed method to reduce the system costs.*

### INTRODUCTION

Shunt capacitors are widely used in distribution networks for voltage adjustment, energy loss reduction, power factor correction and increasing the available capacity of the feeders. Therefore, the optimization problem of determining the size of capacitors and their installation locations considering different loading conditions and the network constraints including load flow, bus voltage and capacitor banks are of primary objectives [1, 2].

In most researches, however, all possible situations in a real distribution network are not considered. For instance, references [3, 4, 5] investigate the capacitor placement problem only under balanced operational conditions. In some other methods, unbalanced conditions are taken into consideration [6, 7, 8]. References [9, 10] develop the optimization techniques for the cases where harmonics and nonlinear loads are present. Considering the network in unbalanced and harmonic conditions is studied in reference [11]. The load variations which are another essential factor in the optimization process are not considered in [12, 13]. The optimal capacitor placement will be at maximum cost effectiveness, if a combination of fixed and switching capacitors can be determined for all load levels, especially medium and peak ones.

In this paper, a fuzzy optimization problem of capacitor placement is presented by using GSA algorithm in the presence of nonlinear loads for radial and meshed distribution networks. In this regard, the system load variations are considered at all three load levels of low, medium, and peak in the presence of harmonic loads. In

the proposed method, sensitive analysis is first used to candidate buses with higher sensitivity for optimal placement of capacitors. Then, the GSA algorithm is used for optimization process so that the optimal location and size of the fixed and switching capacitors are determined. The results presented on the modified IEEE 123-Bus distribution network clearly show the effectiveness of the proposed algorithm.

### PROBLEM FORMULATION

Optimal determination of fixed and switching capacitor locations is one of the reactive power control problems in distribution networks. This is usually performed by considering objective functions and the problem constraints.

#### The proposed objective function

The objective function employed here consists of two parts: energy loss cost and capacitor installation cost. Since fixed and switching capacitors have different prices, this difference should be considered in the optimization process. The installation cost which is the same for both will be also added in the objective function. The energy loss is calculated by the summation of the power losses of each load level multiplied by its time interval. Therefore, the proposed objective function which has to be minimized can be expressed as follows:

$$S = K_e \sum_{j=1}^L T_j P_j + \sum_{i=1}^{ncap} (K_{di} + K_{cf} Q_{cfi} + K_{cs} Q_{csi}) \quad (1)$$

Where:

- $K_e$  : the cost of energy losses (\$/kwh),
- $L$  : number of load levels,
- $T_j$  : time duration for  $j$ th load level,
- $P_j$  : power losses at  $j$ th load level,
- $K_{di}$  : the installation cost of capacitors in  $i$ th bus,
- $K_{cf}, K_{cs}$  : the purchase cost of fixed and switching capacitors,
- $Q_{cfi}, Q_{csi}$  : reactive power of the fixed capacitor and switching banks,
- $ncap$  : number of the candidate buses for capacitor installation.

#### The problem constraints

In addition to the minimization of the objective function, a series of the problem constraints should be also satisfied as follows [14]:

- a) Load flow constraint: This constraint is represented by the following equation.

$$F(x^i, u^{A_i, B_i, C_i}) = 0 \quad (2)$$

where  $u^{A_i, B_i, C_i}$  represents the capacitor sizing vector of each phase of the  $i$ th load level and  $x^i$  is the network state variables such as the bus voltage magnitude and the line currents vectors.

b) Bus voltages constraint: This constraint is expressed in (3).

$$V_{k, \min}^{A_i, B_i, C_i} \leq V_k^{A_i, B_i, C_i} \leq V_{k, \max}^{A_i, B_i, C_i} \quad (3)$$

where  $V_k^{A_i, B_i, C_i}$  is the voltage magnitude of each three phases in  $k$ th bus and  $i$ th load level. The “min” and “max” present the minimum and maximum values of the parameters, respectively.

c) THD constraint: The distortion factor constraint of voltage is considered by specifying maximum total harmonic distortion of bus voltages ( $\text{THD}_i \leq \text{THD}_{\max}$ ).

$$\text{THD}_i = \frac{\sqrt{\sum_{h \neq 1}^{h_m} |V_{i,h}|^2}}{V_{i1}} \quad (4)$$

$\text{THD}_i$  : total harmonic distortion at bus  $i$ ,

$\text{THD}_{\max}$  : maximum allowable harmonic distortion,

$V_i$  : rms voltage at bus  $i$ ,

$V_{i1}$  : fundamental frequency at bus  $i$ ,

$V_{i,h}$  :  $h$ th harmonic voltage at bus  $i$ ,

d) Capacitor banks constraint: The following equations represent the constraints related to the fixed and switching capacitor banks available in the market (in other words, discrete sizes of capacitor banks):

$$u_k^1 = u_k^2 = \dots = u_k^{nt} \leq u_{k, \max} \quad \text{Fixed Type} \quad (5)$$

$$0 \leq u_k^i \leq u_{k, \max}^i \quad \text{Switching Type} \quad (6)$$

where  $u_k^i$  represents the capacitor size installed in  $k$ th bus and  $i$ th load level.

## FUZZY PROBLEM FORMULATION

In the fuzzy domain, each variable is associated with a membership function ( $\mu$ ) which indicates the degree of the membership. Each variable can have a membership value between zero and one and, therefore, these indexes can be compared in this domain. Accordingly, the parameters of the objective function given in (1), the cost of energy losses, the installation cost of capacitors, the network voltage profile, and the THD index are replaced with fuzzy variables in order to compare them.

### Membership function for economic saving cost

The annual saving cost of capacitor placement in  $j$ th load level ( $N_{sj}$ ) in the distribution network can be calculated as:

$$N_{sj} = K_e T_j P_j - K_e T_j P_j^c - \sum_{i=1}^{ncap} (K_{di} + K_{cf} Q_{cfi} + K_{cs} Q_{csi}) \quad (7)$$

where  $P_j^c$  specifies the power losses in  $j$ th load level after compensation, and  $Q_{cfi}$  is the capacitor value in the  $i$ th bus and  $j$ th load level. Since, the annual saving,  $N_{sj}$ , in (7) should be positive, therefore the following equation must be satisfied.

$$x_j = \frac{K_e T_j P_j^c + \sum_{i=1}^{ncap} (K_{di} + K_{cf} Q_{cfi} + K_{cs} Q_{csi})}{K_e T_j P_j} \quad (8)$$

Equation (8) shows that the high value of  $x_j$  represents the small saving resulting from capacitor placement at  $j$ th load level and vice versa. Therefore, the membership function of the saving cost is defined as:

$$\mu_{sj} = \begin{cases} 1 & x_j \leq x_{\min} \\ \frac{x_{\max} - x_j}{x_{\max} - x_{\min}} & x_{\min} < x < x_{\max} \\ 0 & x \geq x_{\max} \end{cases} \quad (9)$$

In this paper, the values of  $x_{\min}$  and  $x_{\max}$  are considered to be equal to 0.5 and 1, respectively. The  $x_{\min}=0.5$  means that if the saving rate ( $x_j$ ) is equal to or more than 50%, then the membership value will be equal to one. Also,  $x_{\max}=1$  means that the membership value is zero if the saving rate ( $x_j$ ) is zero or negative [15].

### Membership function for the maximum deviation in bus voltage

The purpose for this membership function is to minimize the maximum deviation of voltage buses in each load level. Therefore, the variable  $y_j$  is defined for the  $j$ th load level as follows:

$$y_j = \max |V_s - V_{i,j}| \quad \text{for } i = 2, 3, \dots, NB \quad (10)$$

where  $V_{i,j}$  is the voltage magnitude of  $i$ th bus in  $j$ th load level and  $V_s$  is the voltage magnitude of the source bus. By minimization the maximum value of this voltage deviation, the membership function value becomes larger and vice versa.

$$\mu_{V_j} = \begin{cases} 1 & y_j \leq y_{\min} \\ \frac{y_{\max} - y_j}{y_{\max} - y_{\min}} & y_{\min} < y < y_{\max} \\ 0 & y \geq y_{\max} \end{cases} \quad (11)$$

In the proposed method, the values of  $y_{\min}$  and  $y_{\max}$  are considered to be equal to 0.05 and 0.1, respectively. In other words, if the minimum value of the network voltage is larger or equal to 0.95 pu, the membership value will be equal to one. In a similar manner, the  $y_{\max}=0.1$  means that the minimum value of the network voltage is equal to 0.9 pu and when the voltage is smaller than this value, the membership value is considered to be zero [15].

### Membership function related to the THD index of bus voltage

The goal of this membership function is to minimize the maximum voltage deviation of THD value of bus voltage

in each load level. Therefore, the  $z_j$  variable is defined for  $j$ th load level as follows:

$$z_j = \max(\text{THD}_{i,j}) \quad \text{for } i = 2,3,\dots,NB \quad (12)$$

where  $\text{THD}_{i,j}$  and  $\text{THD}_{\max}$  are considered to be the THD value of  $i$ th bus voltage in  $j$ th load level and the maximum permitted of THD value, respectively.

$$\mu\text{THD}_j = \begin{cases} 1 & z_j \leq z_{\min} \\ \frac{z_{\max} - z_j}{z_{\max} - z_{\min}} & z_{\min} < z < z_{\max} \\ 0 & z \geq z_{\max} \end{cases} \quad (13)$$

In this paper, the values of  $z_{\min}$  and  $z_{\max}$  are equal to 0.03 and 0.05 based on the IEEE-519 standard, respectively.

### Fuzzy multi-objective formulation

The three indexes form the objective function with fuzzy variables after fuzzification (see (9), (11) and (13)), the final objective function for the  $j$ th load level can be considered as follows;

$$F_j = W_1 \cdot \mu s_j + W_2 \cdot \mu V_j + W_3 \cdot \mu\text{THD}_j \quad (14)$$

Where  $W_1$ ,  $W_2$  and  $W_3$  are the constant weighting factors for each parameter and are considered to be equal. With regard to the desirable objectives in the distribution system, these factors are specified by the system operator. In a fuzzy optimization the objective function given in (14) for every load level should be maximized so that the system cost is reduced to its minimum value. It should be mentioned that the inverse value of function  $F$  is employed for minimization in the proposed optimization process.

### **GRAVITATIONAL SEARCH ALGORITHM (GSA)**

In this section, a new optimization algorithm, Gravitational Search Algorithm (GSA), based on the law of gravity [16], has been used to solve the problem. In GSA, agents are considered as objects and their performance is measured by their masses. All these objects attract each other by the gravitational force, and this force causes a global movement of all objects towards the objects with heavier masses. Hence, masses cooperate using a direct form of communication, through gravitational force. The heavy masses –which correspond to good solutions – move more slowly than lighter ones, this guarantees the exploitation step of the algorithm.

In GSA, each mass (agent) has four specifications: position, inertial mass, active gravitational mass, and passive gravitational mass. The position of the mass corresponds to a solution of the problem, and its gravitational and inertial masses are determined using its fitness function. In other words, each mass presents a solution, and the algorithm is navigated by properly adjusting the gravitational and inertia masses. By lapse of time, other masses get attracted by the heaviest mass. This mass represents an optimum solution in the search

space. In GSA, search space can be considered as an isolated system of masses. Those masses obey the law of gravity and law of motion [16].

### **GSA ALGORITHM FOR FUZZY OPTIMIZATION PROBLEM**

In this section, the proposed design procedure of fuzzy optimal placement (location and size) of fixed and switching capacitors is discussed. For this purpose, first, by performing the initial load flow, the specifications of the network such as power losses and their annual energy costs as well as buses voltages and THD values are obtained. Then, buses with higher sensitivity in loss reduction and with lower sensitivity in the propagation of harmonically currents are identified for optimal capacitor placement using sensitive analysis [13]. After selecting the candidate buses by sensitive analysis, the objective function and the problem constraints are fuzzified according to (14) and considered as the fitness function. Now, the optimal location and the size of capacitors are obtained by GSA algorithm.

#### Real modeling of capacitors

In this algorithm a list of available capacitors in the market is prepared and then a suitable list considering the purchasing costs, installation, and maintenance is selected. It is also to be noted that the capacitors should not be selected based on a fixed reactive power because it varies when the feeder voltage changes. Therefore, the capacitor modeling should be performed on as fixed impedance.

The growth rate of investment costs for capacitors is also another important factor because from the economical point of view an acceptable design is the one whose productivity is more than the initial investment costs plus the growth rate of initial costs. Therefore, the initial costs ( $F_{it0}$ ) including the purchase, erection, and maintenance costs must be multiplied by the coefficient for capital turnover. That is,

$$F_{it} = F_{it0} \left[ \frac{i_r(1+i_r)^{D_y}}{(1+i_r)^{D_y} - 1} \right] \quad (15)$$

Where  $F_{it}$  is the annual cost of the capacitors,  $i_r$  is the annual rate of growth of money, and  $D_y$  is the lifespan of the project in years [6].

### **SIMULATION RESULTS**

#### Initial data

The modified IEEE 123-Bus distribution network is employed for the evaluation of the proposed method. This network works in unbalanced conditions at the nominal voltage of 4.16kV. The specifications of this network are presented in reference [17]. Different load levels are considered according to Table I.

TABLE I: DIFFERENT LOAD LEVELS AND ITS DURATION TIMES

Load Level (%)	Light (60)	Normal (80)	Peak (100)
Duration (h)	1000	6760	1000

TABLE II: CONSTRAINTS AND PARAMETERS IN SIMULATION PROCESS

Parameter		Value
Range of Variable	Minimum of Fundamental Voltage (pu)	0.95
	Maximum of Fundamental Voltage (pu)	1.05
	Permissible THD (%) (IEEE Std. 519-1992)	5
	Individual Voltage Harmonic (%)	3
Costs	Energy Cost (\$/kwh)	0.06
	Fixed Cost of Capacitor (\$)	1000
	Purchase Cost of Fixed Capacitor (\$/kvar)	3
	Purchase Cost of Switching Capacitor (\$/kvar)	3.2
Other Parameter	Capacity of Each Bank (\$/kvar)	50
	Rate of Growth Investment Cost (%)	4
	Life-Span of Capacitors (year)	20

The 12 nonlinear loads are considered as unbalanced loads such as six-pulse converters, adjustable speed drive (ASD) and variable frequency drive (VFD). The harmonic specifications of these loads along with their

active and reactive powers are presented in reference [18]. The range of variables and parameters used in simulation process are given in Table II.

### The results of the fuzzy optimal placement

The results are analyzed in radial and meshed configurations of the network. In this network, by closing the (54-59) switch, the meshed configuration is generated. The candidate buses for capacitor placement will be {60, 66, 610, 71, 79, 95, 100, 102, 104, 114}.

Due to harmonic considerations, the candidate buses for the installation of capacitors are first determined by sensitive analysis. Then, the optimal location and the size of these capacitors are optimized by GSA algorithm. The selected buses for capacitor placement at different load levels and in two configurations of the network are determined in Table III.

The optimal placement of capacitors at three load levels comprising network losses, maximum harmonic distortion, and the minimum voltage of the buses are presented in Tables IV and V. Also, it can be seen from Table VI the annual saving obtained from capacitor placement for two radial and meshed configurations were 10389.35 and 10066.75, respectively.

TABLE III: OPTIMAL LOCATION AND SIZE OF CAPACITORS

Selected Buses	Radial Configuration			Meshed Configuration		
	Capacitor Size in Light Loading(kvar)	Capacitor Size in Minimum Loading(kvar)	Capacitor Size in Peak Loading(kvar)	Capacitor Size in Light Loading(kvar)	Capacitor Size in Minimum Loading(kvar)	Capacitor Size in Peak Loading(kvar)
60	50	50	50	50	50	50
71	250	300	300	200	300	300
Total Capacitor	300	350	350	250	350	350

TABLE IV: RESULTS OF RADIAL NETWORK

	Load Level 60%		Load Level 80%		Load Level 100%	
	With Capacitor	Without Capacitor	With Capacitor	Without Capacitor	With Capacitor	Without Capacitor
Minimum Voltage	0.980	1.002	0.957	0.983	0.932	0.960
Maximum of THD	3.10%	4.57%	3.48%	4.70%	3.86%	4.80%
Losses	59.405	48.826	108.970	88.899	176.080	145.460

TABLE V: RESULTS OF MESHED NETWORK

	Load Level 60%		Load Level 80%		Load Level 100%	
	With Capacitor	Without Capacitor	With Capacitor	Without Capacitor	With Capacitor	Without Capacitor
Minimum Voltage	0.982	1.000	0.959	0.985	0.935	0.962
Maximum of THD	3.11%	4.58%	3.48%	4.66%	3.87%	4.82%
Losses	34.045	23.820	74.399	55.084	131.770	101.556

TABLE VI: COMPARISON OF THE RESULTS

Network Reconfiguration	Index	Before Capacitor Placement	After Capacitor Placement
Radial	Annual Cost of Energy Losses (\$)	58327.33	47714.59
	Annual cost of capacitors(\$)	---	223.38
	Annual Power Losses (kwh/year)	972122.16	795243.16
	Total Annual Saving (\$)	---	10389.35
Mesh	Annual Cost of Energy Losses (\$)	40125.43	29834.27
	Annual cost of capacitors(\$)	---	224.11
	Annual Power Losses (kwh/year)	668757.16	497237.83
	Total Annual Saving (\$)	---	10066.75

## CONCLUSIONS

In this paper, a new method for optimal placement of capacitor banks for the reduction of the cost of energy losses and improvement of voltage profile using GSA algorithm was presented. In the proposed method, not only the nonlinear loads in the radial and meshed networks were present, but also the harmonic deviation constraint of the voltage was considered. The major advantages of the proposed algorithm can be summarized as follows;

1- The use of switching capacitors at high load levels, results in the loss reduction and improvement of the network voltage profile. The high price of these capacitors compared with the fixed type is also taken into consideration.

2- By increasing the load level, the THD value in the network increases too. This THD increase before compensation is due to having nonlinear loads. However, after compensation, the compensating capacitors play more important roles.

3- In the meshed network configuration, less compensating capacitors are employed and the installation costs of capacitors are accordingly reduced. However, if switching capacitors are used, the cost of capacitors may be more.

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